Agricultural Finance Review

Interest rates and agricultural debt: estimating the marginal interest rate

Charles B. Moss and Jaclyn D. Kropp
Food and Resource Economics Department, University of Florida, Gainesville, Florida, USA

Received 1 November 2023
Revised 6 June 2024
23 December 2024
Accepted 2 January 2025

Abstract

Purpose – While the average cost of debt capital can be calculated from historical financial statement data by dividing the interest paid each year by the total level of debt, this average cost of debt provides little information regarding the true cost of acquiring additional debt capital, and hence, its use is potentially problematic in financial decision-making. This study focuses on the linkage between observed changes in the average interest rates calculated from financial statements (balance sheet and income statement) and the marginal cost of borrowing or the cost of acquiring new debt. Motivated by the capital asset pricing model (CAPM), the marginal cost of capital is modeled as a function of a risk-free interest rate (the return on Moody's Aaa bonds), returns on the S\&P stock index capturing overall market returns and a portfolio of agricultural stocks to represent farm sector-specific risks.

Design/methodology/approach – Using a unique dataset constructed from United States Department of Agriculture (USDA) state-level Financial Performance of the Farm Sector data for the years 1960 through 2003 and state-level Agricultural Resource Management Survey (ARMS) data for the years 2003–2014 and Bayesian methods, we model the observed interest rate as an autoregressive function controlling for changes in debt and key rates of return in the general economy.

Findings – The results indicate that the marginal interest rate is a function of the Aaa corporate bond rate and the stock market. We also find evidence of a negative relationship between returns to a portfolio of agricultural stocks and the marginal interest rate. Overall, the findings suggest that the imputed interest rate frequently misrepresents the marginal cost of debt capital.

Originality/value – Most farm financial datasets allow for the analysis of the farm firm's average interest rate. However, farmers make decisions based on the marginal cost of credit – the interest rate on a newly issued note. This study estimates this marginal interest rate for the 15 states for which the ARMS data are representative for the years 1960 through 2014 and compares the estimated marginal interest rate with the imputed average interest rate.

Keywords Average interest rate, Bayesian, Marginal cost of capital, Monetary policy

Paper type Research paper

1. Introduction

The cost of capital is perhaps the most widely studied topic in the finance literature. Modigliani and Miller (1958) in their seminal article, contend that the cost of capital is important to three groups of economists: financial specialists concerned with ensuring firm growth and survival; managerial economist concerned with capital budgeting decisions; and economic theorists concerned with explaining investment behaviors. In practice, a firm's cost of capital is typically determined by calculating the firm's Weight Average Cost of Capital (WACC) – a forward-looking measure representing the firm's cost of acquiring additional funds to invest in its business (Solomon, 1963; Myers, 1974; Miles and Ezzell, 1980). WACC can be thought of as the marginal cost of acquiring funds, accounting for both the costs of debt and equity acquisition. The cost of capital influences firm value, capital structure, and capital investments. Modigliani and Miller (1958) theory postulates that firm value is maximized when optimal capital structure is achieved, which occurs when the firm's cost of capital is at its minimum. Furthermore, WACC is typically used as the discount rate in firm-level capital budgeting decisions (Miller and Modigliani, 1961).

JEL Classification — Q14, N1

This work is supported by the Research Capacity Fund (Hatch Multistate) program, project award no. 7007993, from the U.S. Department of Agriculture's National Institute of Food and Agriculture and Farm Credit System Coordinating Committee Research Program Award.

Agricultural Finance Review

e-ISSN: 2041-6326
p-ISSN: 0002-1466
DOI 10.1108/AFR-11-2023-0146

Corporate firms, with access to well-functioning equity and debt markets, tend to use a mix of debt and equity to finance their businesses. However, without access to well-functioning equity markets, farms must rely on retained earnings and debt to fund new investments. Yet, observing a farm's cost of debt financing can be difficult. In the finance literature, it is common to estimate the cost of debt as the average or imputed interest rate calculated by dividing the total interest paid by the total debt outstanding (Francis *et al*., 2005; Hyytinen and Pajarinen, 2007; Ylhäinen, 2017; Grau and Reig, 2020). But doing so can either overstate or understate the marginal cost of acquiring additional debt, particularly when prevailing market interest rates are changing rapidly as has been the case in the past few years.

The macroeconomy significantly impacts the agricultural sector. This relationship between agricultural and the macroeconomy rose in prominence in the 1980s, during the farm financial crisis. Schuh (1974, 1976) note the relationship between macroeconomic fluctuations and the exchange rate, which directly affect the export dependent agricultural sector. In addition, increases in the real interest rates contributed to the demise of the agricultural land market during the crisis (Harl, 1990). More recently between 2008 and 2014, the Federal Reserve System (Fed) used quantitative easing to reduce the shock of the Great Recession following the financial crisis of 2007–2008 (see Kuttner, 2018 for a discussion of the dates and relative magnitudes of quantitative easing). Specifically, the Fed purchased government bonds increasing the money supply and putting downward pressure on real and nominal interest rates. Beginning in 2014, the Fed started to unwrap these investments by repurchasing a fraction of the government bonds as old bonds expired. Overall, from 2008 through 2018 the United States experienced a period of low interest rates, low unemployment and, surprisingly, relatively low rates of inflation. While the Fed returned to a period of quantitative easing briefly in response to the COVID pandemic, the Fed shifted from relatively loose monetary policy of quantitative easing during the 2010s to a period of monetary tightening in the 2020s to combat rising inflation. Specifically, the Fed increased the Effective Federal Funds Rate from 0.08% in early March of 2022 to 0.33% in late March of the same year (Federal Reserve Bank of St. Louis, 2024). In December 2024, the effective Federal Funds Rate was 4.58% (Federal Reserve Bank of St. Louis, 2024).

To contextualize the impact that changes to the Federal Funds Rate can have on the cost of debt to borrowers, consider that as the Federal Funds Rate rose, the 30-Year Mortgage Interest Rate increased from 3.22% in January of 2022 to 5.81% in June of that year (Federal Reserve Bank of St. Louis, 2024). In December 2024, this 30-year rate was 6.60% (Federal Reserve Bank of St. Louis, 2024). While the current rate is relatively high in comparison to values since the early 2000s, it is still less than half of the rate experienced in October of 1981, during the Farm Financial Crisis, when the 30-year mortgage rate peaked at 18.53% (Federal Reserve Bank of St. Louis, 2024).

The impacts of the dynamics of interest rates on the cost of capital raise several significant questions for the U.S. farm sector, including agricultural producers and agricultural banks, and have implications for farm policy. Thus, this study examines the effect of general interest rate movements from broader financial markets on the marginal cost of debt capital or the cost of acquiring new debt in the U.S. farm sector. As previously highlighted, most financial datasets, including agricultural datasets such as the Agricultural Resource Managerial Survey (ARMS), do not facilitate the estimation of the marginal cost of debt capital. Typically, these datasets provide the total interest paid and total level of debt. The interest rate, or the average interest rate on the total debt, is then determined by dividing the interest paid by the total level of debt. Even when loan-level data are provided, it is still difficult to determine the effective interest rate on new borrowing. To further complicate the scenario, the imputed average interest rate is endogenous. For example, if interest rates are declining, like they were between 2008 and 2015, firms may choose to refinance their loans, or bundle old debt with new borrowing to reduce the average interest rate. However, when interest rates are rising, firms hold on to older debt – perhaps paying the minimum on older debt while paying additional principal on more recent debt that was taken out at higher interest rates. Thus, measures of interest rate constructed from financial statement data may misstate the firm's true marginal cost of

borrowing. In periods of increasing interest rates, the average interest rate computed from financial statement data understates the cost of acquiring new debt (the marginal cost of debt capital) with implications for capital budgeting and leverage decisions. In 2008, the 30-Year Mortgage Rate stood at 6.03% (Federal Reserve Bank of St. Louis, 2024) while the imputed average interest rate on agricultural debt was 10.47% (computed from the Agricultural Resource Management Survey's Farm Business Balance Sheet data for the U.S. as a whole by dividing the interest paid by the total liabilities). By 2020, the 30-Year Mortgage Rate had fallen to 3.11% (Federal Reserve Bank of St. Louis, 2024) while the imputed average interest rate on agricultural debt had declined to 5.58%. We use the 30-Year Mortgage Rate to highlight potential differences between the cost of borrowing and imputed average interest rates since real estate loans account for 48.4% of farm debt (computed from the Agricultural Resource Management Survey's Farm Business Balance Sheet data).

While there are numerous articles in the finance literature that estimate the cost of capital, the majority of these studies investigate the cost of equity capital for publicly traded firms using extensions of the Sharpe-Lintner (Sharpe, 1964; Lintner, 1963) capital asset pricing model (CAPM) such as those proposed by Fama and French (1992). In addition to accounting for firm size, many of these studies focus on the impacts of disclosure and accounting transparency on the cost of capital and the cost of debt capital (e.g. Diamond and Verrecchia, 1991; Easley and O'hara, 2004; Lambert *et al*., 2007). However, estimating the marginal cost of capital for agriculture is inherently more difficult than estimating the cost of equity or the cost of debt for publicly traded firms since there is no secondary market for equity at the farm-level. While yield to maturities on Farmer Mac securities may approximate the cost of agricultural debt in a secondary market, Farmer Mac was formed in 1987 following the farm financial crisis and hence the data series is relatively short limiting its usefulness. For publicly traded firms, equity returns can be observed from changes in stock prices and the marginal cost of debt can be inferred from the prevailing yield to maturities on corporate bonds of the issuer or those issued by competing firms with similar credit ratings. The few studies that investigate the cost of equity for private firms suggest a private firm risk premium (Benninga *et al*., 2005; Abudy *et al*., 2016). Furthermore, Martin and Clapp (2015) argue that, historically, the state plays an important role in mediating agricultural finance; due to risk, private capital is reluctant to invest in agriculture without assurances from the state. The Federal Farm Loan Act of 1916, Federal Land Banks, and the Agricultural Credits of 1923 are examples of the United States government taking steps to reduce agricultural producers cost of borrowing and mitigate the risk of agricultural lending by banks (Farm Credit Administration, 2014). Such government programs alter the cost of capital for agriculture. Hence, understanding the relationship between average interest rates calculated from financial statements and the marginal costs of capital to agriculture is important.

In this study, we estimate the marginal cost of borrowing based on the imputed average interest rate calculated from income statement and balance sheet data using a Bayesian approach. As reviewed by Bessler *et al*. (2010), Bayesian econometrics has a long history in agricultural and resource economics. Our use of Bayesian techniques is largely motivated by previous studies that use Bayesian techniques in the estimation of time series models. Specifically, the Bayesian approach allows the estimation of specifications where the number of parameters is large relative to the number of observations, such as the case in our study. Following the discussion of Zellner (1985), the growth in popularity of less structured vector time series models such as the vector autoregression formulation of Sims (1980) led to an explosion in parameters in the standard time-series specification. For example, a vector autoregression with five endogenous variables requires 25 parameters for each lag. If four lags are needed to appropriately specify the time series representation, this implies 100 estimated parameters. Further, the researcher's decision to reduce the number of lags from four could be viewed as an ad hoc restriction and imposes a severe prior on the time series representation. One alternative to imposing this severe restriction is to use Bayesian specifications to estimate longer lag lengths. Doan *et al*. (1983) provide such an example; they estimate a longer lag length model based on a random walk prior for each endogenous variable. Dorfman and

Lastrapes (1996) can also be viewed in this line of literature. Specifically, Dorfman and Lastrapes (1996) use a more comprehensive technique than that proposed by Doan (1990) and Sims (1980) to estimate the impulse response functions for crop and livestock prices to macroeconomic shocks. To avoid the implied dramatic prior on lag-lengths, we use a Bayesian approach that allows us to start with a longer lag length than would be feasible for our dataset using other techniques. This relationship between the lag lengths and the appropriateness of Bayesian methods is further justified by the relatively small sample size. Furthermore, Bayesian techniques can be easier to implement than standard economic estimators in certain situations; for example, Dorfman's (1996) Bayesian specification of the multinomial probit model.

Specifically, we estimates the marginal interest rate for the 15 states for which ARMS data are representative (Arkansas, California, Florida, Georgia, Illinois, Indiana, Iowa, Kansas, Nebraska, North Carolina, Texas, Washington and Wisconsin) as a function of the Aaa bond rate and rates of return of the stock market and a portfolio of agricultural stocks. Our model is motivated by the CAPM; the inclusion of the return on Moody's Aaa bonds represents the risk-free rate, returns on the S&P stock index capture overall market returns, and the portfolio of agricultural stocks captures farm sector specific risks and returns. As expected, we find the marginal cost of debt is significantly impacted by movements in the general prevailing market interest rate represented by the return on Moody's Aaa portfolio. The relationship between stock returns and the marginal cost of debt is less robust – we find a negative relationship for one state, positive relationship for three states, and insignificant relationship for the remaining 11 others states. Consistent with our expectations, we find some evidence that the marginal cost of borrowing decreases as returns on agricultural stocks increases. This suggests that as the agricultural sector becomes more profitable the cost of borrowing by agricultural producers declines, likely as a result of lower probability of delinquency and default. To disentangling the effects of the independent variables from the time-series dynamics, we construct the elasticity of the marginal interest rate with respect to the Aaa bond rate, S&P 500 returns, and returns on a portfolio of agricultural stock. We find substantial heterogeneity by state in these elasticities. Collectively, our results indicate the imputed interest rate frequently misrepresents the marginal cost of acquiring additional debt capital.

In the next section, we present a discussion of the dynamics between debt, interest rates, and balances sheet information. In section 3, we discuss the econometric specification. Section 4 discusses the data. Section 5 presents the results. In Section 6, we concluded with a discussion of the findings and implications.

2. Debt, interest rates, and balance sheet dynamics

The objective of this paper is to estimate the marginal cost of debt for debt issued at time t. Intuitively, we cannot observe the marginal interest rate for the agricultural sector from the income statements and balances sheets. Thus, we recover an approximation of the marginal cost of borrowing from a sequence of accounting statements and other information from financial markets.

The imputed or average interest rate can be observed from the financial statement data. Specifically, we define $\overline{r}_t$ (the imputed or average) interest rate paid on outstanding debt as

$$\overline{r}_t = \frac{R_t}{D_t} = \sum_{i=0}^{m} \frac{D^*_{t-i} r_{t-i}}{D_t} \tag{1}$$

where R_t is the total interest expense paid by the firm at time t, D_t is the total debt outstanding at time t, D^*_{t-i} is the amount of debt issued at time $t-i$ that is still outstanding at time t, and r_{t-i} is the interest rate on debt issued at time $t-i$.

To provide insight into the approximation of the marginal cost of borrowing, consider how the marginal cost of debt at time t is embedded in the imputed interest rate. For our discussion, consider the scenario where the number of lagged debt terms (m) is equal to three

$$r_t = \overline{r}_t - (\alpha_{0,1} r_{t-1} + \alpha_{0,2} r_{t-2} + \alpha_{0,3} r_{t-3})$$
$$\alpha_{0,i} = \frac{D^*_{t-i}}{D^*_i} \tag{2}$$

where the 0 subscript in $\alpha_{0,i}$ denotes the approximation of the marginal interest rate at time $t-0$. The basic problem in directly estimating the marginal interest rate at time t using Equation (2) is that we only observe $\overline{r}_t$. We cannot observe the lagged marginal interest rates (i.e. r_{t-1}, r_{t-2}, and r_{t-3}). One possible solution is to consider a recursive formulation. For example, we could substitute the implicit result for Equation (2) at time $t-1$

$$r_{t-1} = \overline{r}_{t-1} - (\alpha_{1,1} r_{t-2} + \alpha_{1,2} r_{t-3} + \alpha_{1,3} r_{t-4}) \tag{3}$$

back into Equation (2) to yield

$$r_t = \overline{r}_t - \alpha_{0,1}\overline{r}_{t-1} - [(\alpha_{0,2} - \alpha_{0,1}\alpha_{1,1}) r_{t-2} + (\alpha_{0,3} - \alpha_{0,1}\alpha_{1,2}) r_{t-3} + \alpha_{1,3} r_{t-4}]. \tag{4}$$

Note that the number of unobserved lagged interest rates remains the same.

As an alternative, consider the scenario where we model the marginal interest rate using a structural model for factors affecting the marginal interest rate paid on farm debt at each point in time. As a starting point, consider the model

$$\overline{r}_t = \psi_t \widehat{r}_t + \psi^*_{t-1} \overline{r}_{t-1} \tag{5}$$

where ψ_t is the share of the outstanding debt originated at time t that remained unpaid at the end of the year, ψ^*_{t-1} is the share of outstanding debt issued in or before period $t-1$ that remains unpaid at time t, and $\widehat{r}_t$ is an estimated marginal or incremental interest rate for debt issued during period t (i.e. $\psi_t + \psi^*_{t-1} = 1$). Conceptually, our basic assumption is that if we know the factors driving the marginal interest rate at time t, we could use the specification in Equation (5) to create the estimated marginal interest rate based on the change in the average interest rate

$$\widehat{r}_t(\alpha, \tilde{r}_t) = \frac{\overline{r}_t - \psi^*_{t-1} r_{t-1}}{\psi_t}. \tag{6}$$

where $\widehat{r}_t(\alpha, \tilde{r}_t)$ is some function of a set of parameters α and information on current returns $\tilde{r}_t$. Next, we substitute the $\overline{r}_{t-1}$ implied by Equation (5) back into Equation (5) to yield

$$\overline{r}_t = \psi_t \widehat{r}_t + \psi_{t-1} \widehat{r}_{t-1} + \psi^*_{t-2} \overline{r}_{t-2} \tag{7}$$

where ψ_{t-1} is the share of debt issued in the time period $t-1$ that remains outstanding, $\tilde{r}_{t-1}$ is the interest rate paid on debt originated at time $t-1$, and ψ^*_{t-2} is the total share of debt issued in or before time period $t-2$ that remains outstanding.

Next, we subtract the lagged average interest rate from each side of Equation (7) and rewrite $\overline{r}_t - \psi^*_{t-2}\overline{r}_{t-2}$ as a time series process

$$(1-\beta(L))\overline{r}_t = \psi_t \widehat{r}_t + \psi_{t-1}\widehat{r}_{t-1} \tag{8}$$

where $\beta(L)$ is some general time-series process. We can generalize this specification based on the current value of the marginal interest rate and m lags as

$$(1-\beta(L))\overline{r}_t = \sum_{i=0}^{m} \psi_{t-i}\widehat{r}_{t-i} \tag{9}$$

For our current purposes, we will set $\beta(L)$ as a third-order autoregressive function and $m = 5$ (i.e. the specification includes six marginal interest rates for each observation – $\tilde{r}_t$ through $\tilde{r}_{t-5}$). While these appear to be potentially constraining assumptions, the empirical results suggest a fairly low order for the time series approximation (i.e. the estimated values of β_2 and β_3 are seldom statistically significant). Further, given that our sample contains 55 observations for each state, limiting the number of marginal interest rates (i.e. $m + 1$) to six implies that we are estimating 8 parameters with 49 observations.

The next question is how to specify our representation for the marginal interest rate for each year. As a starting point, we propose

$$\widehat{r}_t = \alpha_0 + \alpha_1 \tilde{r}_t^{Aaa} + \alpha_2 \tilde{r}_t^{S\&P} + \alpha_3 r_t^{Ag\ Stock} \tag{10}$$

where $\tilde{r}_t^{Aaa}$ is the interest rate on Aaa bonds, $\tilde{r}_t^{S\&P}$ is the market return on the Standard and Poor's 500 stock portfolio, and $\tilde{r}_t^{Ag\ Stock}$ is the return on a portfolio of agricultural stocks. This formulation can be weakly linked to a traditional CAPM formulation (Lintner, 1965; Sharpe, 1964) that has been extended to account for industry specific returns. Specifically, the return on stock k is modeled as

$$r_{kt} = \alpha_k + \beta_k\left(r_{mt} - r_t^{rf}\right) + \gamma_k z_t + \nu_t \tag{11}$$

where α_k is the excess return, β_k is the return on risk relative to the market (r_m), r_t^{rf} is the risk-free rate of return, $\gamma_k z_t$ is a measure of industry or structural risk, and ν_t is a residual. In a typical CAPM formulation, the researcher is interesting in testing for a market equilibrium where $\alpha_k = \gamma_k = 0$. In our application, we are interested in using the market information to recover the marginal cost of borrowing, not to test for market equilibrium between the marginal interest rate in the agricultural sector and the overall capital market.

Combining the above assumptions into a single specification, our empirical model becomes

$$\begin{aligned}\overline{r}_t = \beta_0 + \beta_1\overline{r}_{t-1} + \beta_2\overline{r}_{t-2} + \beta_3\overline{r}_{t-3} + \\ \sum_{i=0}^{5}\psi_{t-i}\left(\alpha_0 + \alpha_1\tilde{r}_{t-i}^{Aaa} + \alpha_2\tilde{r}_{t-i}^{S\&P} + \alpha_3\tilde{r}_{t-i}^{Ag\ Stocks}\right) + \epsilon_t\end{aligned} \tag{12}$$

However, since we cannot observe ψ_{t-i}, we will use $\tilde{\psi}_{t-i} = (D_{t-i} - D_{t-i-1})/D_t$ as a proxy for ψ_{t-i}.

One artifact of modeling anything using a time-series process (i.e. $(1-\beta(L))\overline{r}_t = \alpha_0 + \alpha_1 x_t + \epsilon_t$) is disentangling the effect of the independent variable from the time-series dynamics. In this study, we propose an approximation that creates a steady-state representation of the time series specification around time period t

$$\overline{r}_t = \frac{\beta_0 + \left(\sum_{t=0}^{5}\tilde{\psi}_i\right)\left(\alpha_0 + \alpha_1\tilde{r}_t^{Aaa} + \alpha_2\tilde{r}_t^{S\&P} + \alpha_3\tilde{r}_t^{Ag\ Stocks}\right)}{1-\beta_1-\beta_2-\beta_3} \tag{13}$$

where $\tilde{r}_t^{Aaa}$, $\tilde{r}_t^{S\&p}$ and $\tilde{r}_t^{Ag\ Stocks}$ are smoothed values using a cosine kernel for five years around t (see Appendixes A2 for details). Using the projected marginal interest rate, we then compute the elasticity of the marginal interest rate with respect to the Aaa bond rate as

$$\eta_t = \frac{\alpha_1\sum_{i=0}^{5}\psi_i}{1-\beta_1-\beta_2-\beta_3}\frac{\tilde{r}_t^{Aaa}}{\widehat{r}_t}. \tag{14}$$

(see details in Appendixes A2). One potential difficulty of this formulation is that the elasticity's sign depends on $\sum_{i=0}^{5}\tilde{\psi}_i$. Specifically, if the change in debt over the last five years is positive and α_1 is positive (the normal case) then this elasticity is positive. Hence, an increase in the Aaa bond return leads to an increase in the marginal interest rate. However, if $\sum_{i=1}^{5}\tilde{\psi}_i$ is less than zero as is the case when the level of debt is declining, then elasticity becomes negative. We follow a similar procedure to construct the elasticities with respect to changes in S&P 500 returns and returns to the portfolio of agricultural stocks.

Our model highlights the endogenous nature of the imputed average interest rate; it is a function of both the marginal cost of debt and farmers' financing/refinancing decisions which are influenced by the marginal cost of debt. We hypothesize that the marginal cost of debt or marginal interest rate is a function of movements in financial markets. Specifically, we expect the relationship between the Aaa bond rate and marginal cost of acquiring additional debt to be positive; as the prevailing market interest rate increases, the marginal cost of borrowing should also increase. Furthermore, we expect the marginal interest rate to increase with S&P 500 returns; an increase in S&P 500 returns represent an increase in the opportunity cost of lending and hence agricultural lenders will require a higher interest rate to be paid by farmers as S&P 500 returns increase. Finally, we expected a negative relationship between the marginal interest rate and the returns on our portfolio of agricultural stock; an increase in the relatively profitability of the agricultural sector implies a lower risk of delinquency and default and hence agricultural lenders will charge lower interest rates in time when the sector is more profitable.

3. Econometric specification

The reemergence of Bayesian techniques in econometrics is related to changes in empirical macroeconomics. Sims (1980) questioned whether large structural macroeconomic models adequately represented the dynamic relationships in the economy. In response, he pioneered the use of smaller vector autoregressions to analyze the impact of monetary and fiscal policy. However, an important point in this development was lack of parsimony. For example, a quarterly model of five variables required five parameters for each equation for each additional lag (e.g. a lag of five quarters in a quarterly model implies 25 parameters). This result was compounded by the need to estimate the covariance matrix across equations. One response was the rise of Bayesian vector autoregession (Doan *et al.*, 1983) which used Theil mixed estimators to increase the number of lags which could be estimated. The argument was that "feasible" lag specifications implied rather stringent specification restrictions. In addition, recent advances in Bayesian techniques have significantly extended the specifications that can be estimated. With techniques such as BUGS and HMC, fairly innovative likelihood functions and prior distributions can be incorporated.

We estimate the parameters in Equation (12) using the Stan Development Team (2022) implementation of the Bayesian sampler. MC-Stan differs somewhat from Bayesian implementations that use Gibbs Samplers (e.g. BUGS) that have become popular in agricultural economics. Specifically, MC-Stan uses a Hamiltonian Markov Chain (HMC)

sampler. We provide a brief sketch of the HMC sampler in Appendix B. We apply the HMC sampler using RStan. We bound the roots of the time series function to be between −0.95 and 0.95 which imposes stationarity on the autoregressive function. We use the default settings for the remaining coefficients which assumes a quadratic approximation of the normal density function (see the discussion of Equation (B.5) in Appendix B). Finally, the estimation for each state is based on 40,000 draws of 6 chains. In addition, we thin the posterior to every 50th draw to reduce any autocorrelation from the Markovian process. This formulation gives us 2,400 observations in the posterior distribution for each state.

4. Data

We combine data from two USDA datasets – the Financial Performance of the Farm Sector and Agricultural Resource Management Survey (ARMS) - to construct an annual state-level panel dataset from 1960 through 2014 [1]. From 1960 through 2003, the USDA published the state-level Financial Performance of the Farm Sector for the 48 contiguous states. More recent data on the farm financial position for 15 states are available through the Agricultural Resource Management Survey (ARMS). Constructing a state-level panel of farm financial characteristics requires linking these two datasets. In this study, we are interested in three series available in both data series – acres per farm (acrepfarm), debt per farm (debtpfarm), and interest paid per farm (intpefarm). Each of these data are either directly available or can be easily computed based on data provided. Financial Performance of the Farm Sector report state-level aggregate data, while the ARMS is a farm-level survey of a sample of farmers. Hence, adjustments are required to make the estimated values comparable. To facilitate the comparison, we convert the Financial Performance of the Farm Sector data to farm-level data by averaging across farms (i.e. we transform the aggregate data to data for a representative farm firm). We limit our analysis to prior to 2015 due to changes in the treatment of the household residence and associated liabilities in later years of ARMS data.

A challenge in combining the two series to construct the panel dataset is that there is only one year common to both samples – 2003. Further, there is some question regarding the quality of this observation (e.g. unlike other years in the data, this data point is perpetually preliminary in the state-level Financial Performance of the Farm Sector datasets). However, the two data sets overlap at the aggregate (or 48 state-level) between 1996 and 2003. Thus, we use three computations to derive an adjustment coefficient: the coefficient for the single overlap year, the average across state-level overlaps for 2003, and the aggregate overlap for 1996 through 2003. Defining the overlap coefficient (ζ_{it}) as

$$\zeta_{it} = \ln\left(\frac{\tilde{v}_{it}}{v_{it}}\right) \tag{15}$$

where $\tilde{v}_{it}$ is the value from the ARMS sample and v_{it} is the value from the state level panel, we derive the state level value for the 2003 comparison as

$$\zeta_{i,2003} = \ln\left(\frac{\tilde{v}_{i,2003}}{v_{i,2003}}\right), \tag{16}$$

where $i = 1, \ldots, 15$, the average 2003 comparison as

$$\overline{\zeta}_{i,2003} = \frac{1}{15}\sum_{i=1}^{15} \zeta_{i,2003}, \tag{17}$$

and the normalization across the ARMS states

$$\overline{\zeta}_{i,\cdot} = \frac{1}{13}\sum_{t=1996}^{2003} \ln\left(\frac{\tilde{v}_{US,t}}{v_{US,t}}\right). \tag{18}$$

The aggregate estimate is then computed as

$$\widehat{\zeta} = (1-\kappa_1-\kappa_2)\zeta_{i,2003} + \kappa_1\overline{\zeta}_{i,2003} + \kappa_2\overline{\zeta}_{i,\cdot} \tag{19}$$

where κ_1 and κ_2 are parameters selected to bound the forward gap between the observed U.S. aggregate and the aggregated values from the ARMS dataset.

The rate of return data for stocks are obtained from the Center for Research on Security Prices (CRSP) accessed through the Wharton School of Business, University of Pennsylvania (2024). CRSP directly provides the return on the Standard and Poor's 500 (S&P 500) portfolio. We constructed an agricultural stock portfolio to estimate whether agricultural stocks produce a risk premium or discount relative to the overall S&P index. The basic contention is that the risk premium associated with agricultural corporations can affect the marginal cost of agricultural debt because the returns to the agricultural and agribusiness sectors are functions of many of the same factors. We constructed this portfolio by first searching for codes for standard industrial classification systems associated with agricultural activities (Table A1). Searching using these codes yielded 757 possible stocks. Next, we restricted our set of possible stocks to those stocks that were continuously traded between January 1960 and December 2014 for consistency and to avoid the complexities of constructing an index using an inconsistent set of stocks. This resulted in a set of 14 stocks. However, we dropped six stocks from this list. We dropped Texas Instruments since its primary focus is no longer agricultural. International Harvester was dropped because it traded under bankruptcy toward the end of our sample. Universal Leaf was dropped because it is now a company focused on vaping products. DWG (Wendy's) was dropped because it is "fast food" company and hence its returns are unlikely to be correlated with production agriculture. Similarly, we also dropped Coca Cola and Pepsi hypothesizing that they share few command factors with other agribusiness stocks. After dropping these stocks, we were left with a set of eight stocks in our portfolio (Archer Daniel Midlands, Tootsi Roll Industries, Hersey, General Mills, FMC Corporation, Deere & Company, Campbell Soups, and Kellogg Co). We constructed the rate of return for our index by weighting the returns for each stock by market capitalization for each stock.

Finally, the Moody Aaa bond rate was obtained from the Federal Reserve Bank of St. Louis's online database (Federal Reserve Bank of St. Louis, 2024). Specifically, we used FRED's achieved Moody's seasonally adjusted Aaa corporate bond yield.

5. Results and discussion

The Bayesian estimates of the parameters in Equation (12) for each state are presented in Table 1. As expected, the first-order autoregression coefficient is significant for every state ranging from 0.9801 for Indiana to 0.7727 for North Carolina. The results reject non-stationarity for every state but Texas (i.e. the hypothesis that $1-\beta_1-\beta_2-\beta_3$ is statistically different than zero is rejected).

The coefficients on the Aaa bond rates (α_1) are statistically significant for thirteen of the fifteen ARMS states. This effect ranges from 1.2404 in Wisconsin to 0.3581 in Missouri and 0.3370 in Texas (neither of the latter two is statistically significant at any conventional confidence levels). Hence, the results support the conjecture that the interest rate paid in the farm sector increases as the Aaa bond rate increases. This result would support an equilibrium between the debt market in the farm sector and the corporate bond market.

The results for the effect of changes in the returns to the general stock market on the interest rate paid by the farm sector (α_2) are less robust. Consistent with our expectations, an increase in the return on S&P 500 stocks is associated with an increase in the marginal interest rates for

Table 1. Parameter estimates for change in interest rate over time

Parameter	Arkansas	California	Florida	Georgia	Illinois	Indiana	Iowa	Kansas
α_0	−0.0190	−0.0226*	−0.0079	−0.0580***	−0.0565***	−0.0389*	−0.0914***	−0.0280
	(0.0171)[a]	(0.0126)	(0.0094)	(0.0164)	(0.0187)	(0.0221)	(0.0194)	(0.0200)
α_1	0.3597*	0.4192***	0.2387*	0.8873***	0.6898***	0.5358**	1.1726***	0.5030**
	(0.1928)	(0.1544)	(0.1383)	(0.2154)	(0.2141)	(0.2503)	(0.2278)	(0.2557)
α_2	0.0597	−0.0703**	0.0428	0.0696	0.0902**	0.0580	0.1496***	−0.0075
	(0.0515)	(0.0350)	(0.0433)	(0.0531)	(0.0417)	(0.0427)	(0.0323)	(0.0434)
α_3	−0.0709	0.0002	−0.0474	−0.1477***	−0.0524	−0.0636	−0.1671***	−0.0545
	(0.0565)	(0.0455)	(0.0392)	(0.0444)	(0.0483)	(0.0444)	(0.0442)	(0.0525)
β_0	0.0122	0.0094	0.0009	0.0385***	0.0247***	0.0169	0.0435***	0.0144
	(0.0093)	(0.0060)	(0.0043)	(0.0095)	(0.0086)	(0.0108)	(0.0089)	(0.0097)
β_1	0.9536***	0.8511***	0.8597***	0.9374***	0.9525***	0.9801***	0.9013***	0.9249***
	(0.1118)	(0.0972)	(0.0808)	(0.1005)	(0.1118)	(0.1112)	(0.0915)	(0.1039)
β_2	−0.0077	0.0488	0.1173**	−0.1715***	−0.0773	−0.0421	−0.1915***	−0.0136
	(0.0711)	(0.0613)	(0.0547)	(0.0617)	(0.0613)	(0.0688)	(0.0576)	(0.0660)
β_3	−0.1116	−0.0412	−0.0158	−0.2169***	−0.1606*	−0.1407*	−0.2107***	−0.0930
	(0.0841)	(0.0498)	(0.0351)	(0.0804)	(0.0828)	(0.0848)	(0.0736)	(0.0686)
σ_ϵ^2	0.0063***	0.1413**	0.0388	0.0068***	0.0053***	0.0053***	0.0045***	0.0055***
	(0.0007)	(0.0702)	(0.0573)	(0.0008)	(0.0006)	(0.0006)	(0.0005)	(0.0006)
$1-\beta_1-\beta_2-\beta_3$	0.1656**	0.0051***	0.0065***	0.4511***	0.2853***	0.2027**	0.5009***	0.1817**
	(0.1019)	(0.0006)	(0.0007)	(0.1055)	(0.0933)	(0.1152)	(0.0969)	(0.1051)

	Minnesota	Missouri	Nebraska	North Carolina	Texas	Washington	Wisconsin
α_0	−0.0385*	−0.0195	−0.0217	−0.0299*	−0.0161	−0.0387***	−0.1099***
	(0.0233)	(0.0193)	(0.0218)	(0.0167)	(0.0199)	(0.0118)	(0.0297)
α_1	0.5955**	0.3581	0.6086**	0.6213***	0.3370	0.5670***	1.2404***
	(0.2560)	(0.2180)	(0.2475)	(0.1826)	(0.2256)	(0.1513)	(0.2968)

(*continued*)

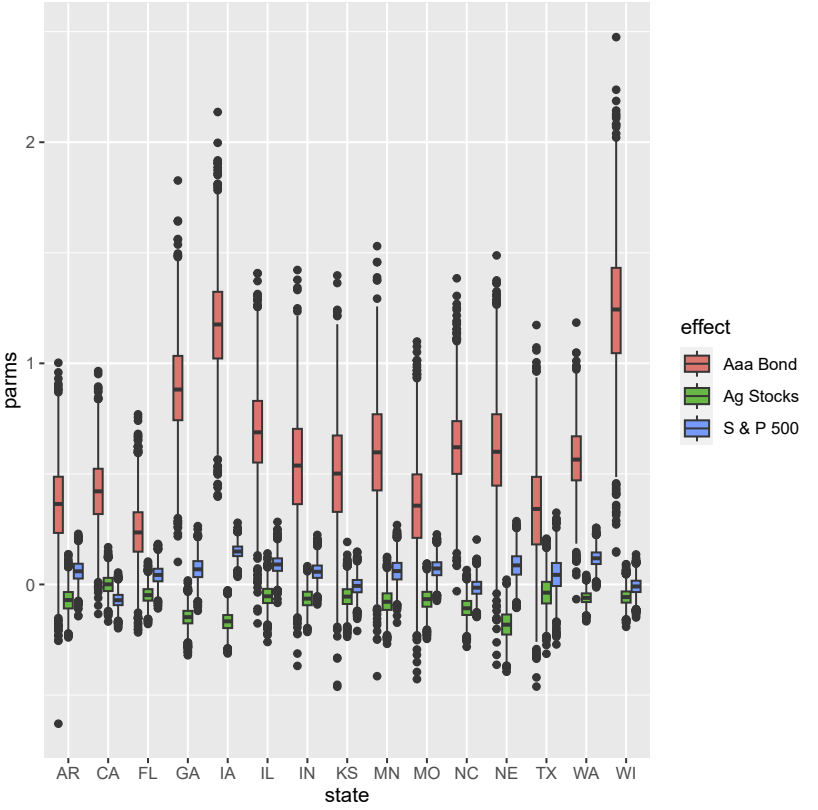
Table 1. Continued

	Minnesota	Missouri	Nebraska	North Carolina	Texas	Washington	Wisconsin
α_2	0.0606	0.0712	0.0861	−0.0148	0.0448	0.1184***	−0.0096
	(0.0565)	(0.0447)	(0.0633)	(0.0408)	(0.0786)	(0.0403)	(0.0399)
α_3	−0.0777	−0.0674	−0.1817***	−0.1064**	−0.0373	−0.0602**	−0.0565
	(0.0555)	(0.0515)	(0.0649)	(0.0477)	(0.0715)	(0.0292)	(0.0397)
β_0	0.0166	0.0104	0.0119	0.0155**	0.0056	0.0169***	0.0598***
	(0.0107)	(0.0098)	(0.0095)	(0.0079)	(0.0080)	(0.0062)	(0.0148)
β_1	0.8892***	0.9485***	0.9054***	0.7727***	0.8752***	0.8218***	0.8887***
	(0.1108)	(0.1129)	(0.1013)	(0.0932)	(0.0927)	(0.0925)	(0.0925)
β_2	−0.0208	0.0077	−0.0171	0.0258	0.0661	0.0023	−0.2878***
	(0.0792)	(0.0725)	(0.0701)	(0.0684)	(0.0662)	(0.0653)	(0.0883)
β_3	−0.0869	−0.0996	−0.0850	−0.0322	−0.0315	−0.0454	−0.2751***
	(0.0766)	(0.0803)	(0.0729)	(0.0409)	(0.0426)	(0.0495)	(0.0908)
σ_ϵ^2	0.0064***	0.0054***	0.0058***	0.0061***	0.0071***	0.0052***	0.0049***
	(0.0007)	(0.0006)	(0.0006)	(0.0007)	(0.0008)	(0.0006)	(0.0006)
$1-\beta_1-\beta_2-\beta_3$	0.2185**	0.1433*	0.1967**	0.2337***	0.0902	0.2214***	0.6742***
	(0.1165)	(0.1059)	(0.1024)	(0.0886)	(0.0911)	(0.0713)	(0.1595)

Note(s): [a] Numbers in parenthesis denote standard error of the estimates, *** denotes statistical significance at the 0.01 level of confidence, ** denotes statistical significance at the 0.05 level of confidence, and * denotes statistical significance at the 0.10 level of confidence
Source(s): Table created by authors

Illinois, Iowa and Washington. However, we find that an increase in the return on S&P 500 stocks is associated with a decrease in the marginal interest rate in California. The lack of consistent effect of changes in the stock market returns may indicate that changes in relative risk between the lending and stock markets have already been taken into account in the bond market, and hence markets are efficient. The effect of changes in the returns on agricultural stocks (α_3) is significant for five states. For these states, an increase in the return on agricultural stocks is associated with a decrease in the marginal interest rate. This suggests that when the agricultural sector is more profitable the cost of borrowing is lower, presumably because the risk of delinquency and default are lower when agriculture is experiencing a boom.

The relative amount of information is further substantiated by box and whisker plot of the posterior distribution for each parameter presented in Figure 1. This depiction is similar to the results presented in Table 1 which relies on asymptotic normality, but allows for some degree of nonnormality. The results support the significance of the effect of Aaa bonds on the marginal interest rate paid in the farm sector in that the lower quarter is never below zero. Further, all the data points are greater than zero for Georgia, Iowa and Wisconsin. In addition, the lower quartile for the effect of general stock market returns (α_2) is greater than zero for Arkansas, Georgia, Iowa, Indiana, Minnesota, Missouri, Nebraska, and Washington. However, the quartile of the effect of the general stock market is less than zero for California, Florida, and Wisconsin. Finally, the upper quartile for the effect of agricultural



Source(s): Figure created by authors

Figure 1. Comparison of the effect of the financial market

stocks (α_3) is less than zero except in the case of California. Hence, a more nonparametric examination of the results support the overall contention that increases in the Aaa bond rates lead to an increase in the marginal interest rate paid in the farm sector while increases in the general rate of return on stocks increases the marginal interest rate. The effect of an industry specific risk effect is fairly small, but consistently negative. In general, the quartile band is fairly symmetric – consistent with normality, but there is some evidence of outliers.

Figures 2 through 5 present the observed interest rate computed from the financial statements, the estimated marginal cost of acquiring additional debt, and error for each state. Collectively, these results suggest that the estimated model predicts the average interest rate well. Each specification picks up the boom nominal interest rates between 1970 and 1980 peaking in 1985. The predicted interest rate then declines rather steadily throughout the sample. Several states including Arkansas, California, Georgia, Illinois, Indiana, Kansas, Minnesota, North Carolina, Texas, Washington, and Wisconsin pick up the increased interest rate from 2007 through 2012.

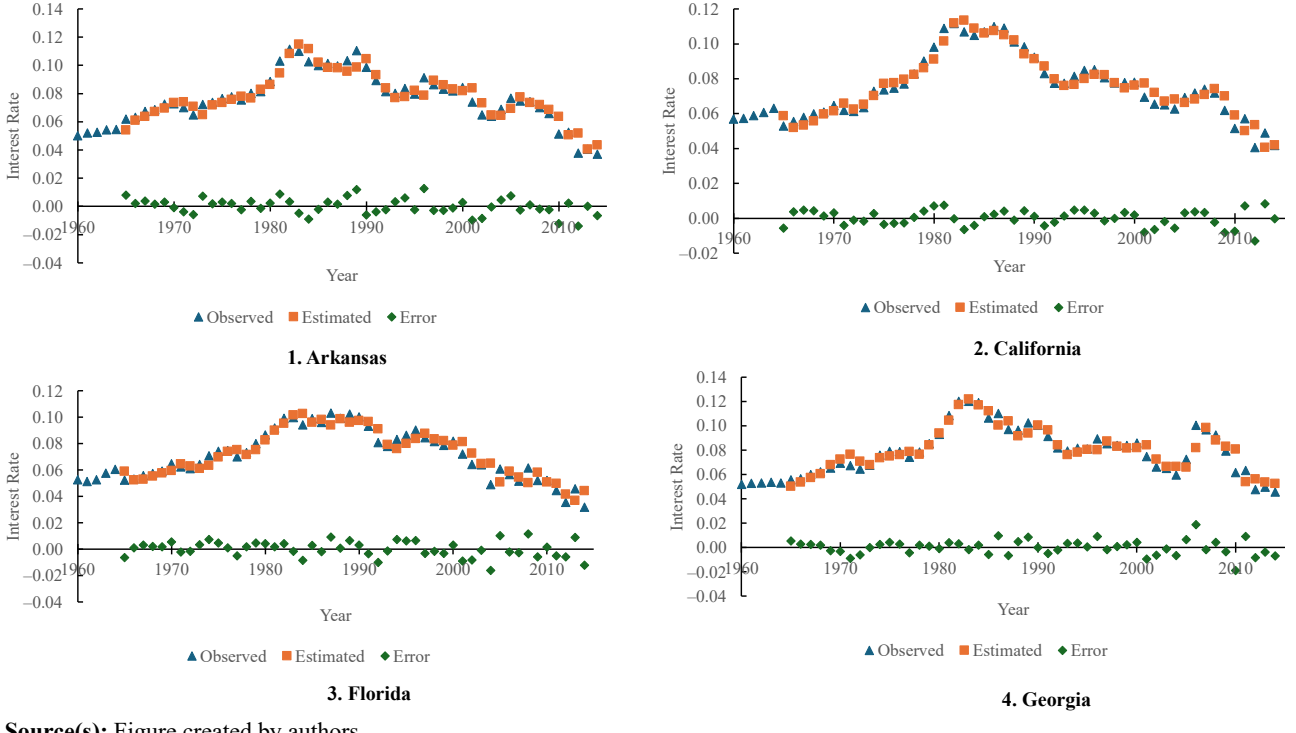
Figures 6 through 9 present the projected marginal interest rate as derived in Equation (13) with the observed interest rate computed using each state's income statement and balance sheet. These results highlight the lead/lag relationships implicit in Equation (12). Specifically, the results for California, Georgia, Illinois, Indiana, Iowa, Kansas, and Minnesota show how the increase in the marginal interest rate from 1970 through 1980 led to the general rise in the observed interest rate. In each of these states, the marginal interest rate tends to reach a maximum in 1980 through 1982. Following this maximum, the observed interest rates follow increasing to peak in 1982 through 1984.

Following the peak of the marginal interest rate in the early 1980s, it declines across states through the mid-1990s. The observed interest rates follow this decline to around 2000. Most of the states also show a bump (a slight increase) in the marginal interest rate with the onset of the great recession between 2006 and 2010, followed by a sharp decline with the onset of quantitative easing following the great recession.

These stylistic results are a little more problematic for some states. For example, the results for Arkansas presented in Figure 6(1) show the same general cycle for both the projected marginal interest rate and the observed interest rate, but the amplitude of the projected marginal interest rate is too large. This larger amplitude of may be explained by how close these time series solution are to non-stationarity (i.e. $1 - \beta_1 - \beta_2 - \beta_3$ the denominator in Equation (13)). As presented in Table 1, $1 - \beta_1 - \beta_2 - \beta_3$ is 0.1656 which is smaller than most other states. The results for Florida show the same amplitude problem (Figure 6(3)) with an estimated value for the stationarity statistic of 0.0065, and Missouri (Figure 8(10)) with a stationarity result of 0.1433. Of course, California presents a contra-example. The relationship between the marginal interest rate and the observed interest rate are similarly scaled despite the fact that California's stationarity measure is 0.0051.

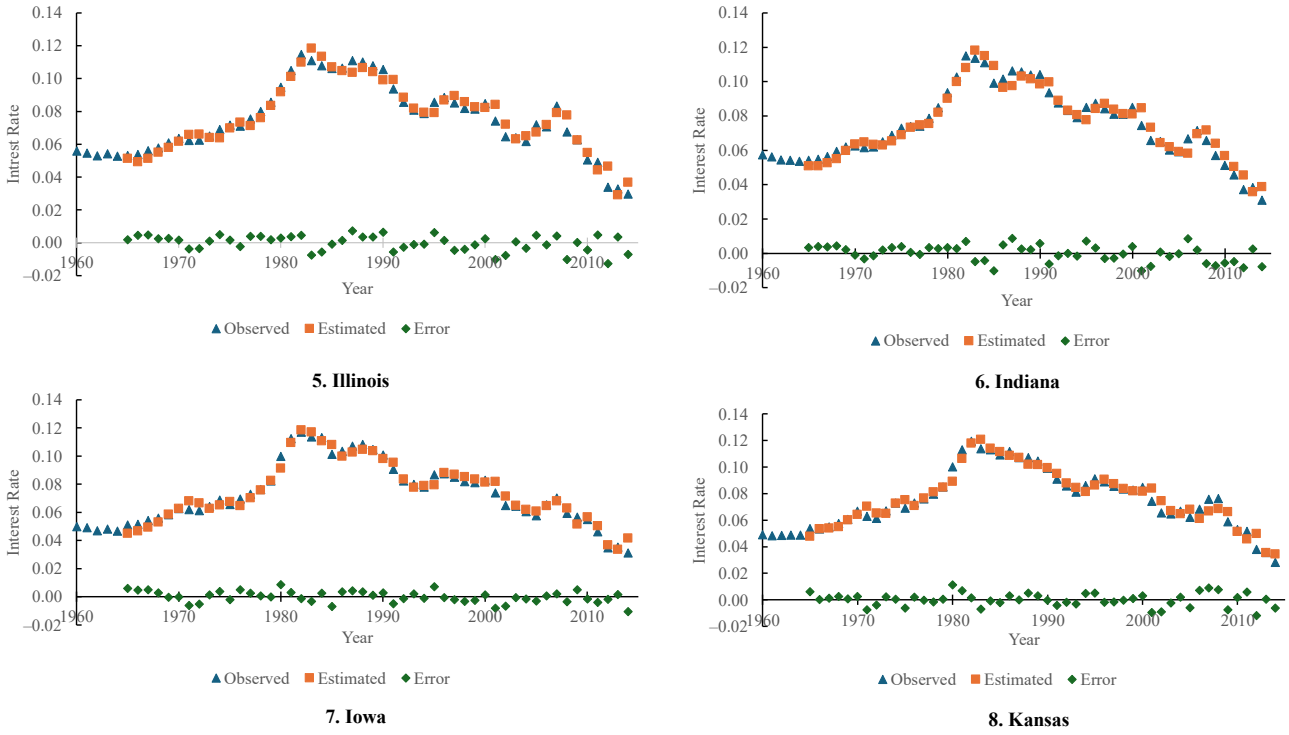
Table 2 presents the estimated elasticities of the marginal interest rate with respect to changes in the Aaa bond rate. The implicit statistical hypothesis reported in Table 3 is the standard construct of whether the parameter value equals zero. Following this concept, the elasticity of the marginal interest rate is statistically significant for Georgia in 1970, 1980, 1990, 2000, and 2010, Illinois in 2970, 1980, and 2010, Iowa in all years, Washington in 1980, and Wisconsin in 1970, 1980, 1990, and 2010.

An alternative hypothesis merits consideration. Specifically, an alternative hypothesis could be whether the estimated elasticity equals 1.0, or whether the marginal interest rate follows the Aaa bond rate. The hypothesis that this elasticity equals one is rejected for Georgia in 1990, 2000, and 2010, Iowa in 1980, 1990, 2000, and 2010, and Wisconsin in 1990 and 2010. These results are likely driven by factors such as a declining level of debt in the case of Georgia in 1990 and 2010, Iowa in 1990, and Wisconsin in 1990. In the remaining cases, the elasticity is less than one. For example, the estimated elasticity in Georgia in 2000 is 0.38490; hence, the marginal interest rate appears to be less responsive to the bond market than expected.



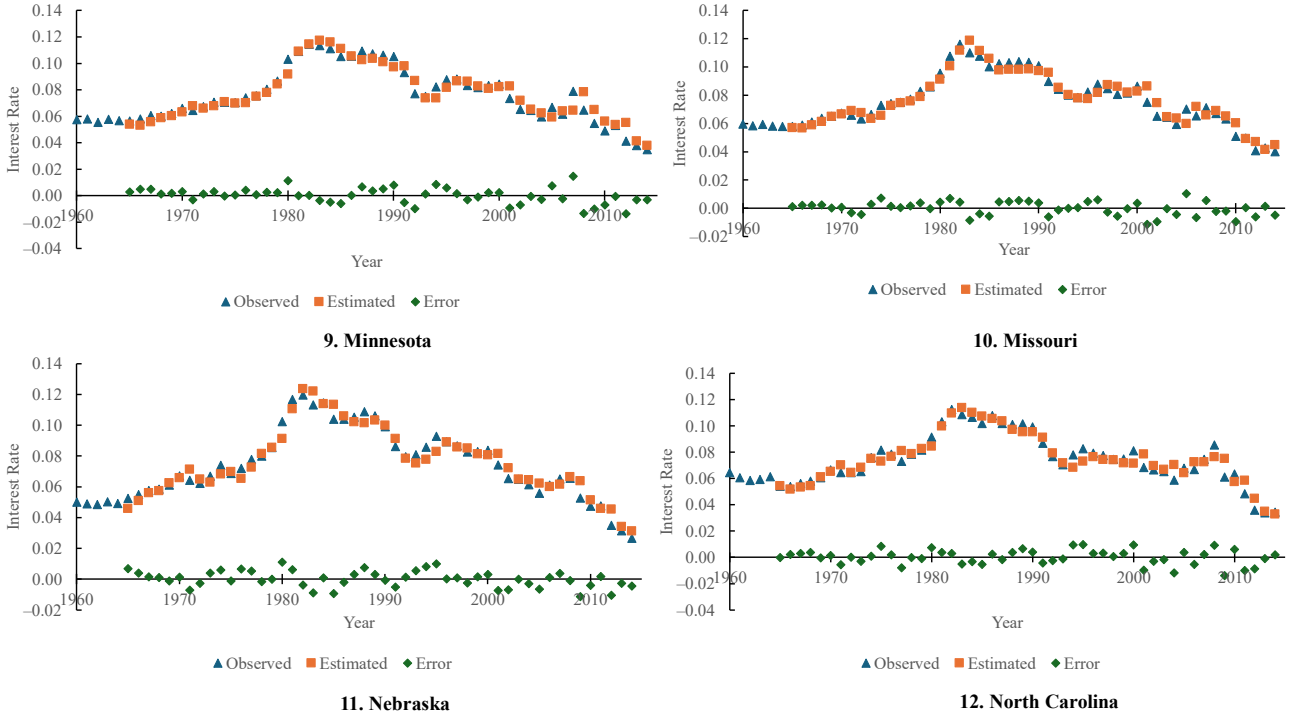
Source(s): Figure created by authors

Figure 2. Predicted versus actual interest rates for Arkansas, California, Florida, and Georgia



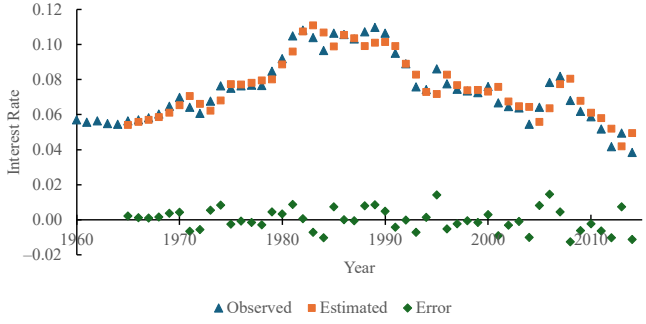
Source(s): Figure created by authors

Figure 3. Predicted versus actual interest rates for Kansas, Illinois, Indiana, and Iowas

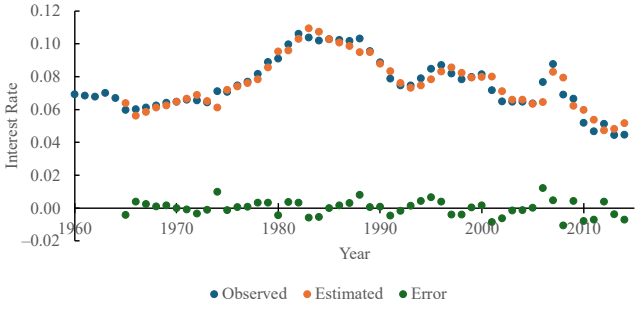


Source(s): Figure created by authors

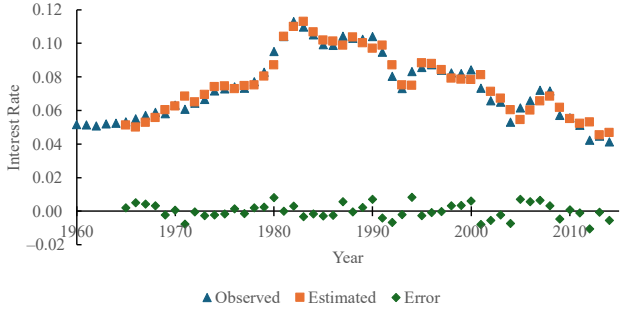
Figure 4. Predicted versus actual interest rates for Minnesota, Missouri, Nebraska, and North Carolina



13. Texas



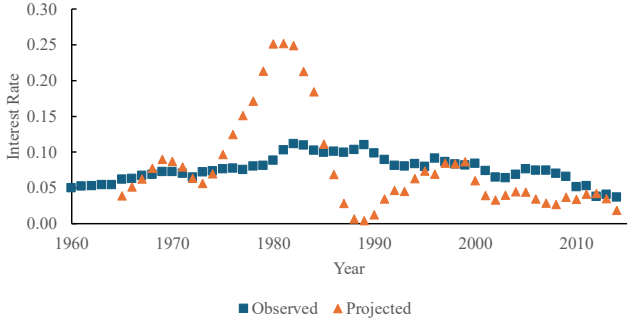
14. Washington



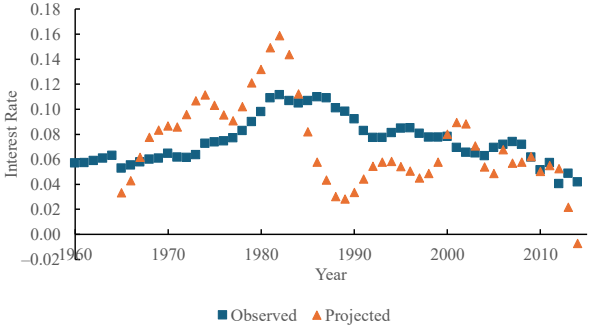
15. Wisconsin

Source(s): Figure created by authors

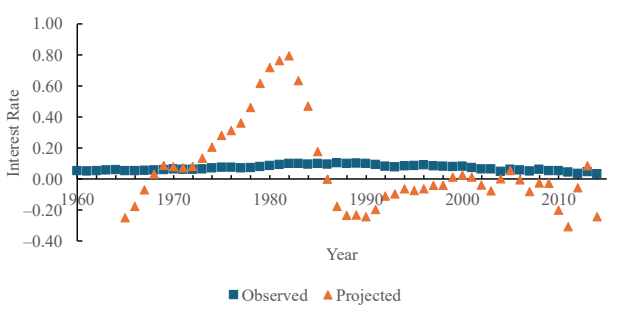
Figure 5. Predicted versus actual interest rates for Texas, Washington, and Wisconsin



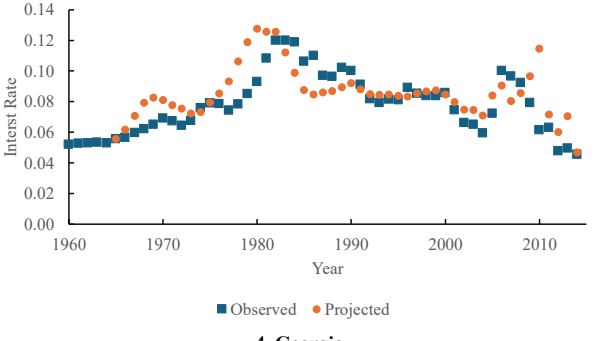
1. Arkansas



2. California



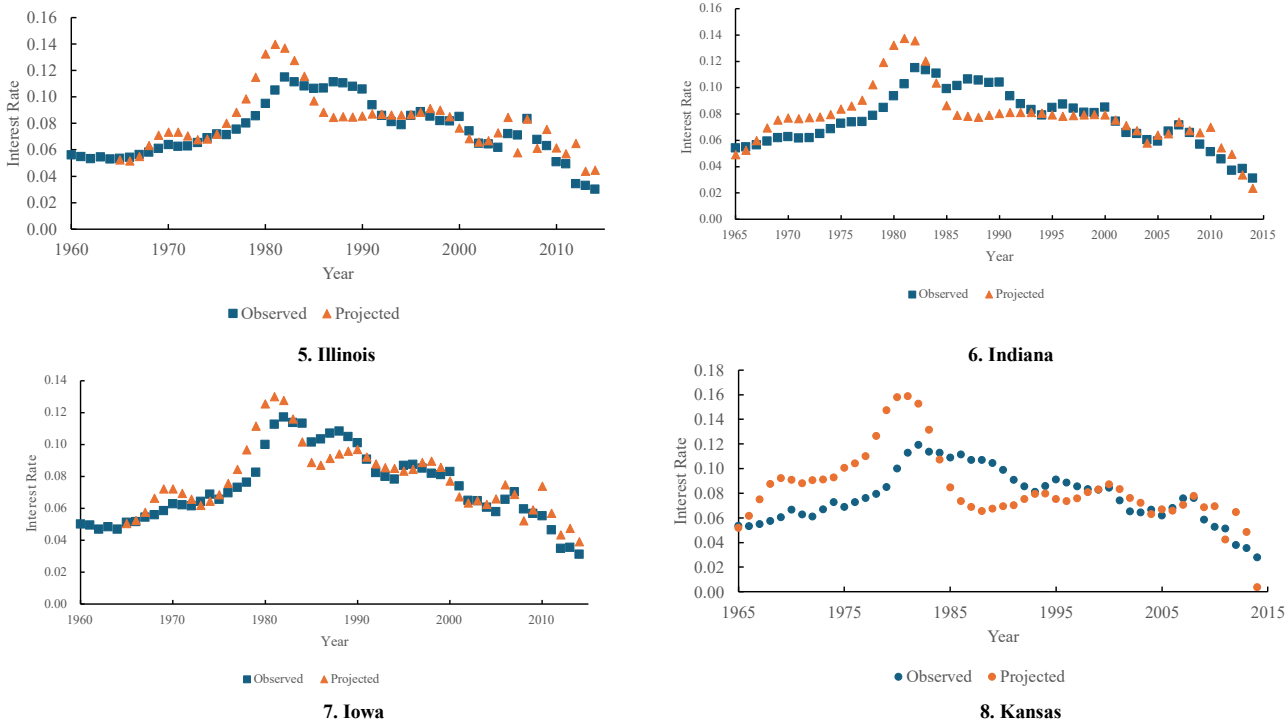
3. Florida



4. Georgia

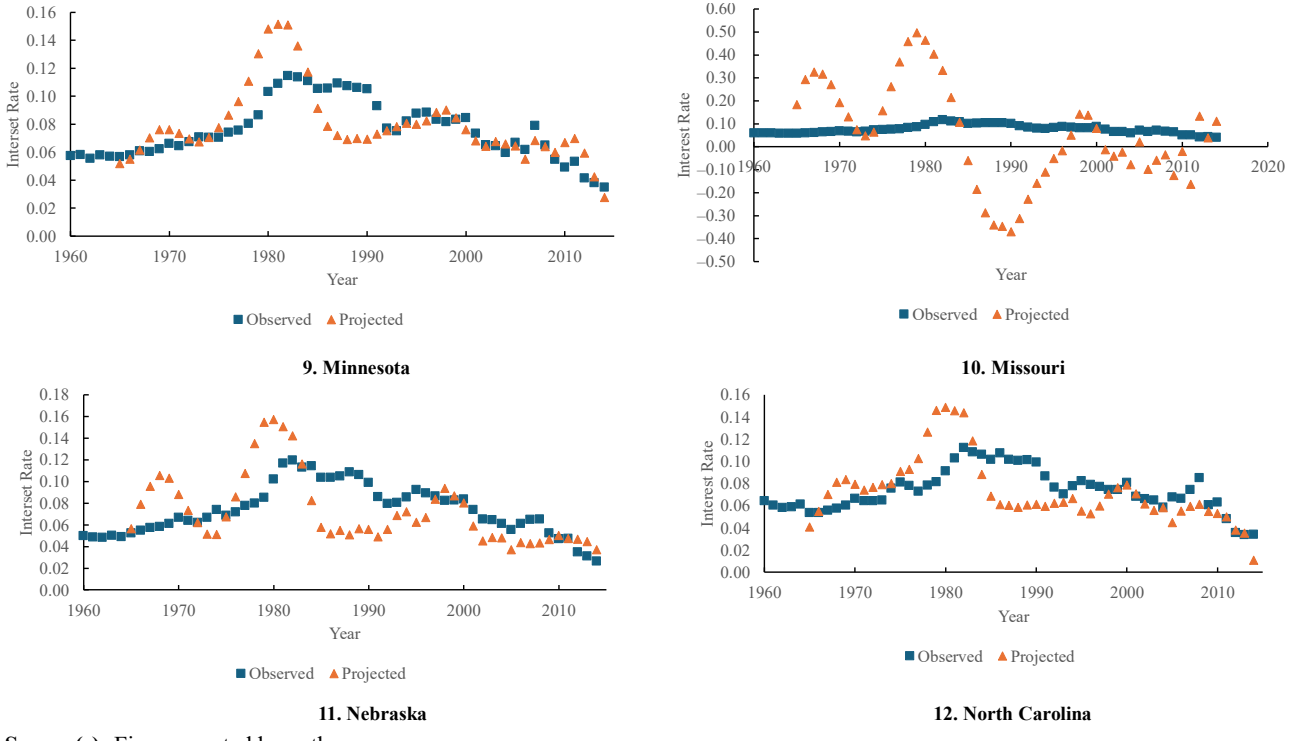
Source(s): Figure created by authors

Figure 6. Normalized variation in predicted interest rate Arkansas, California, Florida, and Georgia



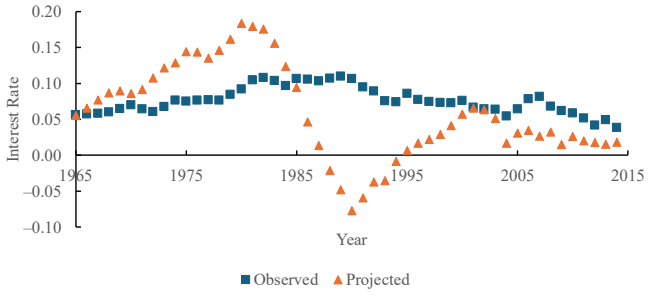
Source(s): Figure created by authors

Figure 7. Normalized variation in predicted interest rate Illinois, Indiana, Iowa, and Kansas

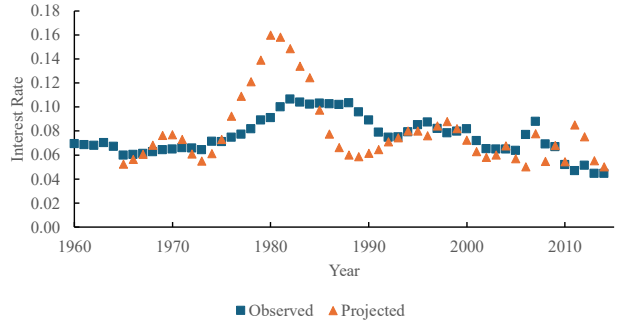


Source(s): Figure created by authors

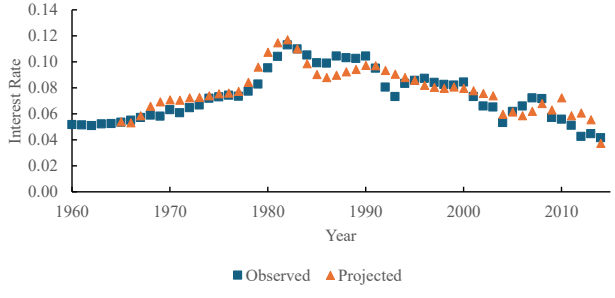
Figure 8. Normalized variation in predicted interest rate Minnesota, Missouri, Nebraska, and North Carolina



13. Texas



14. Washington



15. Wisconsin

Source(s): Figure created by authors

Figure 9. Normalized variation in predicted interest rate Texas, Washington, Wisconsin

Table 2. Elasticity of marginal interest rate with respect to Aaa bond rate

State	Year 1970	1980	1990	2000	2010
Arkansas	1.83529	2.12557	−0.19215	0.56288	0.61868
	(62.80072)[a]	(51.98894)	(7.91419)	(7.50264)	(6.99327)
California	1.42882	1.23887	−2.20992	0.73275	0.83099
	(15.39882)	(1.58951)	(46.04000)	(2.09386)	(8.35359)
Florida	2.76430	−0.17563	0.54409	0.60780	0.43189
	(55.83968)	(99.04234)	(72.35042)	(44.51726)	(28.53372)
Georgia	0.87313***	1.13662***	−0.88769***	0.38490***	−0.43089***
	(0.15490)	(0.12638)	(0.20807)	(0.08910)	(0.07790)
Illinois	0.96238***	1.25102***	−0.83043	0.49796	0.56949*
	(0.37310)	(0.31099)	(1.21870)	(1.18270)	(0.32595)
Indiana	1.09201	1.36703	−0.31170	0.47697	−1.06240
	(2.38735)	(1.47298)	(19.76869)	(3.79827)	(78.89815)
Iowa	0.98094***	1.30005***	−0.93171***	0.57256***	0.21757***
	(0.09911)	(0.07393)	(0.12420)	(0.08003)	(0.02721)
Kansas	1.04009	0.35148	−1.16066	0.46358	0.42445
	(1.06401)	(37.73890)	(23.42737)	(310.90283)	(4.52030)
Minnesota	1.38102	1.29854	−10.58442	0.77578	0.32793
	(3.72314)	(2.41192)	(579.47892)	(7.46500)	(8.10011)
Missouri	−1.65929	1.17030	−1.34809	0.89349	0.39204
	(120.38956)	(6.75517)	(27.76756)	(5.16377)	(3.97191)
Nebraska	2.06075	1.34785	−2.33205	0.74115	1.32390
	(39.40299)	(1.29899)	(73.86453)	(34.56710)	(17.87367)
N Carolina	1.19244	1.48477	−1.57519	0.66909	−2.59738
	(0.73189)	(0.55681)	(38.86635)	(1.49244)	(154.87094)
Texas	1.06753	2.62849	1.62390	0.60329	0.01045
	(7.02107)	(81.50952)	(82.21758)	(20.20133)	(1.64062)
Washington	1.24288	1.11451***	−0.81205	0.35252	0.97172
	(2.17032)	(0.19242)	(5.03256)	(0.89763)	(11.66948)
Wisconsin	0.84135***	1.17793***	−0.61704***	0.38374	0.24045**
	(0.10798)	(0.06905)	(0.17093)	(0.52024)	(0.11195)

Note(s): [a] Numbers in parenthesis denote standard error of the estimates, *** denotes statistical significance at the 0.01 level of confidence, ** denotes statistical significance at the 0.05 level of confidence, and * denotes statistical significance at the 0.10 level of confidence
Source(s): Table create by authors

Table 3 presents the elasticity of the marginal interest rate with respect to changes in the S&P 500 stock returns. In general, these results have far fewer statistically significant estimates. The results for Iowa are statistically significant across all years. In 1970, 1980, and 2010, the estimated values are positive, if somewhat small. Hence, an increase in the rate of return on the S&P 500 is associated with an increase in the marginal interest rate paid by agriculture. This result is consistent with our expectations. Specifically, as the return on the S&P 500 increase, the opportunity cost of lenders increases, and they will only lend to agriculture at a higher interest rate.

Finally, Table 4 presents the elasticity of the marginal interest rate with respect to a change in returns on our agricultural stock index. These estimates are typically statistically significant for Georgia and Iowa; however, the results in 1970, 1980, 2000, and 2010 are negative which is somewhat inconsistent with the conjecture that the return on agricultural stocks would proxy an agricultural risk premium. As an alternative, we propose that the return to agricultural stocks may measure the profitability of the farm sector. In this case, an increase in the return on agricultural stocks would be associated with an increase in the profitability of the farm sector which could imply that the rate returns on agricultural assets have increased. If the rate of

Table 3. Elasticity of the marginal interest rate with respect to a change in the S&P return

State	Year 1970	1980	1990	2000	2010
Arkansas	−0.03529	2.44423	−0.10156	−0.00440	0.05548
	(3.67210)[a]	(111.57530)	(2.03829)	(1.63354)	(1.35562)
California	−0.19210	−0.18533	0.91756	0.05294	−0.10157
	(3.95186)	(0.57764)	(24.34637)	(0.46900)	(1.47123
Florida	0.45325	0.26530	0.70694	0.00874	0.21932
	(13.92057)	(5.63890)	(39.39525)	(6.21303)	(5.66136)
Georgia	0.02402	0.06563	−0.08751	−0.00977	−0.02246
	(0.01892)	(0.05117)	(0.07022)	(0.01725)	(0.01794)
Illinois	0.04721	0.12931	−0.14538	−0.03072	0.05363
	(0.03723)	(0.10877)	(0.48275)	(0.23118)	(0.07517)
Indiana	0.04876	0.16203	−0.01678	−0.03665	−0.20812
	(0.30823)	(2.34325)	(3.13793)	(0.41720)	(10.77036)
Iowa	0.04564***	0.12798***	−0.15761***	−0.02564***	0.01943***
	(0.00942)	(0.02466)	(0.03908)	(0.00553)	(0.00461)
Kansas	−0.01735	0.23376	0.07724	0.00570	−0.01315
	(0.23032)	(15.81839)	(3.05089)	(0.26484)	(0.61837)
Minnesota	0.04097	0.09969	−8.11771	−0.02832	0.04287
	(0.31848)	(0.83929)	(392.90435)	(0.24039)	(1.46674)
Missouri	−0.58959	0.20522	−0.31086	−0.04465	0.04353
	(33.99085)	(1.52996)	(7.27644)	(1.46961)	(0.73370)
Nebraska	0.07909	0.13941	−0.16793	−0.00258	0.13210
	(0.95698)	(0.75328)	(12.35903)	(1.72148)	(2.33681)
N Carolina	−0.00924	−0.03101	−0.08284	0.00497	−0.15860
	(0.04127)	(0.10398)	(4.69520)	(0.02561)	(7.91342)
Texas	0.06709	−0.56188	0.42983	−0.02300	0.00356
	(0.98599)	(37.81887)	(25.17706)	(1.10457)	(0.42828)
Washington	0.09564	0.18177***	−0.22650	−0.02627	0.17003
	(0.16941)	(0.06510)	(1.86264)	(0.03228)	(2.68479)
Wisconsin	−0.00247	−0.00783	0.00599	0.00099	−0.00126
	(0.01136)	(0.03283)	(0.03934)	(0.00604)	(0.00879)

Note(s): [a] Numbers in parenthesis denote standard error of the estimates, *** denotes statistical significance at the 0.01 level of confidence, ** denotes statistical significance at the 0.05 level of confidence, and * denotes statistical significance at the 0.10 level of confidence
Source(s): Table created by authors

return on agriculture increases, the repayment risk on agricultural lending declines. As a robustness check for the effect of agricultural stocks, we compare the results using our index of agricultural stocks to the results using only Deere/Company to measure agribusiness risk. Overall, these results are very similar with a negative, but statistically insignificant effect of effect of return on this stock for most states. These results are available from the authors upon request.

In examining the implications for our estimates for individual states, certain balance sheet dynamics become important. Specifically, changes in the average interest rate are the result of two changes through time – variations in the marginal cost of capital and changes in the level of borrowing. While our usable sample contains data from 1965 through 2014, most of the systematic variation occurs in the period 1975 through 1989 (e.g. the inflation period of the 1970s through the farm financial crisis of the 1980s). Given that our specification relates changes in the average interest rate to implicit changes in the cost of capital through changes in borrowing, this period is very important to our study. Most states increased borrowing significantly from 1972 through 1982. However, with the onset of the farm financial crisis, the debt level declined across states reaching a low in 1987 through 1989 (depending on the state).

Table 4. Elasticity of the marginal interest rate with respect to a change in Ag stock returns

State	Year 1970	1980	1990	2000	2010
Arkansas	0.52481	−3.09707	0.16290	−0.03137	−0.32941
	(32.28624)[a]	(137.17398)	(6.81215)	(0.28071)	(5.54157)
California	0.17484	0.02328	−0.33500	−0.00113	0.00873
	(5.29980)	(0.34053)	(8.10281)	(0.29628)	(1.68046)
Florida	−1.43073	−0.31764	−1.50251	−0.00895	−0.66466
	(43.44132)	(8.87805)	(92.53800)	(4.58746)	(17.53584)
Georgia	−0.15063***	−0.21348***	0.39680***	−0.01559	0.13373***
	(0.04058)	(0.05405)	(0.11574)	(0.01226)	(0.04057)
Illinois	−0.07425	−0.10009	0.15167	−0.01008	−0.07914
	(0.08971)	(0.14925)	(0.38367)	(0.01980)	(0.15097)
Indiana	−0.15081	−0.32962	−0.01139	0.01963	0.46294
	(1.28947)	(5.76582)	(7.82614)	(2.16635)	(26.11596)
Iowa	−0.14566***	−0.20894***	0.36259***	−0.01991***	−0.05812***
	(0.03632)	(0.04841)	(0.10539)	(0.00512)	(0.01671)
Kansas	−0.10336	−0.80289	0.45299	−0.01122	−0.04837
	(0.39337)	(34.71421)	(11.02596)	(0.12994)	(1.94511)
Minnesota	−0.17105	−0.20506	12.97283	−0.02019	−0.18440
	(0.92049)	(0.86678)	(722.85248)	(0.33000)	(3.93995)
Missouri	0.59866	−0.28629	0.65853	−0.04523	−0.09954
	(66.75459)	(3.59953)	(16.69978)	(1.44560)	(2.52108)
Nebraska	−0.45319	−0.47987	0.99420	−0.02288	−0.85950
	(3.05043)	(2.41409)	(65.79635)	(2.02459)	(13.87526)
N Carolina	−0.23926	−0.31231	1.23391	−0.03065	1.49590
	(0.62962)	(0.60683)	(43.02906)	(0.05504)	(84.96369)
Texas	−0.17132	0.85842	−0.88061	−0.00162	0.02041
	(2.94589)	(56.98670)	(43.55803)	(0.73838)	(2.46147)
Washington	−0.14150	−0.13427*	0.24125	−0.00956	−0.26684
	(0.26963)	(0.06882)	(2.28749)	(0.01395)	(5.12598)
Wisconsin	−0.04139	−0.06259	0.08099	−0.00455	−0.02133
	(0.03647)	(0.04999)	(0.11009)	(0.00731)	(0.03812)

Note(s): [a] Numbers in parenthesis denote standard error of the estimates, *** denotes statistical significance at the 0.01 level of confidence, ** denotes statistical significance at the 0.05 level of confidence, and * denotes statistical significance at the 0.10 level of confidence
Source(s): Table created by authors

In each case, the ability or willingness to reduce the level of debt interacts with the marginal cost of capital to determine the average interest rate in each state. Specifically, our results indicate that farmers in Illinois reduced borrowing less rapidly than farmers in Kansas between 1982 and 1987. However, it remains unclear whether this was due to differences in credit constraints or farm-level financing decisions. Coupling this result with a higher estimated marginal cost of capital, the average interest rate for Illinois fell less rapidly than in Kansas. Further, the average interest rate for agricultural states fell less rapidly than other interest rates in the economy (e.g. the Aaa corporate bond rate).

While our results are plausible, they raise additional questions. First, why does the debt level fall slowly in agriculture? In our original problem formulation, we expected an asymmetric response to changes in the interest rate, but we expected farmers to attempt to refinance more when the interest rate fell than when it rose. A potential flaw in our model is that we assumed the choice of refinancing was primarily that of the farmer. Some of the observed change in debt positions during the farm financial crisis may also be attributed to foreclosures and debt write downs rather than repayment. Furthermore, during the farm crisis many farmers may have been unable to refinance because of financial difficulties (e.g. low

solvency or stressed cash flow). Variants of these questions have been analyzed in models of asymmetric information such as Innes (1990). Hence, the relative riskiness of agriculture may affect the farmer's ability to refinance. One suggestion for future work is to use a conditional model for the riskiness of agricultural returns (i.e. an autogressive conditional heteroscedasticity [ARCH] model) to measure the effect of these potential credit constraints.

A second point of difficulty involves the transition in the sources of data. Specifically, we shift from the state-level aggregates for 1960 through 2003 to data from the Agricultural Resource Management Survey (ARMS) for 2004 through 2014. Apart from possible changes in the dynamics around the joining point, there is evidence that the variance of the data is higher for the ARMS data than the state-level data. Differences in the variance of the two datasets might be due to different sampling procedures. As additional years of ARMS data become available, the process of joining the two datasets could be improved by putting more weight on the ARMS observations.

A final concern surrounds the relationship between the marginal cost of capital and the average interest rate following 2008. While that fact that the average interest rate declines faster than the marginal rate is likely attributed to outstanding debt being refinance at lower interest rates or rate adjustments on variable rate debt, the fact that the average rate falls below the marginal rate is perplexing. It seems that while it is possible for the marginal cost of capital to be below the average interest rate calculated from balance sheet and income statement figures, this would require the marginal cost of capital to be increasing overtime, which was not the case. One possible explanation for the observed relationship following 2008 is the discontinuity due to the precipitous drop in the prevailing market interest rate as a result of quantitative easing caused the model to perform poorly in 2008. Another possible justification is difficulties in transitioning between the two data sources; as previously stated there is considerably more noise in the ARMS dataset. Yet another possible explanation is our treatment of the intercept term in the model. Implicitly, we assume that the difference between the fluctuations in the true marginal cost of capital and the average interest rate is constant over time. However, this value may be affected by such factors as changes in the structure of banks serving agriculture. For example, the dramatic restructuring of the Farm Credit System following the farm financial crisis of the 1980s may have shifted the margin between the bond market and the cost of capital charged to farmers. Future work may should consider making the constant a function of some structural variable or a function of some structural variable plus a random fluctuation. However, care should be used so that the structural variable is not correlated with other important factors such as the relative risk in the farm sector.

6. Conclusions

The estimation of the cost of capital in agricultural raises several empirical complications. Historical financial data typically allows the analyst to estimate the average cost of debt by simply dividing the interest paid in a given year by the total level of debt. However, this average interest rate is of limited use in analyzing farm financial decisions or the analysis of agricultural policy. The most relevant interest rate for these purposes is undoubtedly the marginal interest rate that must be paid by farmers for new debt. This study estimates this marginal interest rate for 15 states (Arkansas, California, Florida, Georgia, Illinois, Indiana, Iowa, Kansas, Nebraska, North Carolina, Texas, Washington and Wisconsin) for which the ARMS is representative by regressing the average interest rate on the interaction between changes in debt level and financial information observed in the general economy. Our results indicate that the marginal interest rate paid by agriculture changes over time as a function of the Aaa corporate bond rate. To a lesser extent, we find the marginal interest rate paid by agriculture also changes with the return on the Standard and Poor's 500 Portfolio; we find heterogeneity in the relationship between S&P 500 returns and the marginal cost of borrowing by state. Specifically, the relationship between the marginal cost of debt and the S&P 500 is positive for three states, negative for one, and insignificant for the remaining 11. The lack of

significance for the vast majority of states may suggest the relative risk between the lending and stock markets have already been taken into account in the Aaa bond rate. We find a negative relationship between the return on a portfolio of agricultural stocks and the marginal interest rate, suggesting the marginal cost of debt is lower when profits in the sector are higher. This finding suggests that as the rate of return on agriculture increases, the repayment risk on agricultural lending declines, and hence agricultural lenders charge lower interest rates.

The findings have implications for agricultural lending and on-going policy discussions regarding the next farm bill, especially given the current inflation versus economic growth decisions faced by the Fed. Inflation in the United States started to increase as the Fed started to unwind quantitative and direct payments in response to COVID ceased. Between 2021 and 2023, the onset of inflation contributed to the pressure on the Fed to increase the interest rate in an attempt to control inflation. While these efforts have had the desired effect on inflation, they contributed to a significant increase in the market interest rate with the interest rate on Aaa bonds increasing from 2.45% in January of 2021 to 5.61% in October of 2023 (Federal Reserve Bank of St. Louis, 2024). Given these fluctuations, our results suggest that the marginal interest rate in agriculture will increase significantly. In addition, this effect may be compounded by concomitant declines in the stock market.

Our findings also have implications for farm asset values, particularly farmland values which account for approximately 83% of farm asset values (calculated from recent ARMS data). Using standard present value calculations, the value of a farmland parcel is the discount stream of earnings associated with ownership (Doll *et al*., 1983). Thus, as the marginal cost of debt increases, the discount rate increases, and farmland values decrease. Hence, using the imputed average interest rate as the discount rate rather than the marginal cost of acquiring capital can lead to over or understand asset values. Recent research by Basha *et al*. (2021) finds that cuts to the federal funds rate have long-lasting impacts on farmland values. The authors find that it takes over a decade for the effects of an interest rate change to be fully capitalized in farmland values. Moreover the timing and magnitudes of the rate adjustments impact their findings. Similarly, we find asymmetric responses to changes in the financial market conditions.

Notes

1. Miller *et al*. (2024) examine how interest rates charged vary across lenders. Using ARMS farm-level data, their analysis employs loan-level information on the five largest loans taken out by each farm. In general, their results show that there are significant differences in the interest rate charged by different lenders from 2008 through 2022. While loan-level information available in the farm-level ARMS database provides insights into the marginal cost of borrowing over time as macroeconomic conditions change, unfortunately, the ARMS dataset only became available after 1996. Thus, detailed loan-level information about marginal borrowing is unavailable for periods of significant structural changes in agriculture such as the Farm Financial Crisis of the 1980s.

References

Abudy, M., Benninga, S. and Shust, E. (2016), "The cost of equity for private firms", *Journal of Corporate Finance*, Vol. 37, pp. 431-443, doi: 10.1016/j.jcorpfin.2016.01.014.

Basha, A., Zhang, W. and Hart, C. (2021), "The impacts of interest rate changes on us midwest farmland values", *Agricultural Finance Review*, Vol. 81, pp. 746-766, doi: 10.1108/afr-11-2020-0163.

Benninga, S., Helmantel, M. and Sarig, O. (2005), "The timing of initial public offerings", *Journal of Financial Economics*, Vol. 75 No. 1, pp. 115-132, doi: 10.1016/j.jfineco.2003.04.002.

Bessler, D.A., Dorfman, J.H., Holt, M.T. and LaFrance, J.T. (2010), "Econometric developments in agricultural and resource economcis: the first 100 years", *American Journal of Agricultural Economics*, Vol. 92 No. 2, pp. 571-589, doi: 10.1093/ajae/aaq010.

Diamond, D.W. and Verrecchia, R.E. (1991), "Disclosure, liquidity, and the cost of capital", *The Journal of Finance*, Vol. 46 No. 4, pp. 1325-1359, doi: 10.2307/2328861.

Doan, T. (1990), *Users's Manual RATS Version 3.10*, VAR Econometrics, Evanston, IL.

Doan, T., Litterman, R.B. and Sims, C.A. (1983), "Forecasting and conditional projection using realistic prior distributions", NBER Working Paper – 1202.

Doll, J.P., Widdows, R. and Velde, P.D. (1983), "The value of agricultural land in the United States: a research report", *Agricultural Economics Research*, Vol. 35 No. 2, pp. 39-44.

Dorfman, J.H. (1996), "Modeling multiple adoption decisions in a joint framework", *American Journal of Agricultural Economics*, Vol. 78 No. 3, pp. 547-557, doi: 10.2307/1243273.

Dorfman, J.H. and Lastrapes, W.D. (1996), "The dynamic responses of crop and livestock prices to money-supply shocks: a Bayesian analysis using long-run identifying restrictions", *American Journal of Agricultural Economics*, Vol. 78 No. 3, pp. 530-541, doi: 10.2307/1243271.

Easley, D. and O'hara, M. (2004), "Information and the cost of capital", *The Journal of Finance*, Vol. 59 No. 4, pp. 1553-1583, doi: 10.1111/j.1540-6261.2004.00672.x.

Fama, E.F. and French, K.R. (1992), "The cross-section of expected stock returns", *The Journal of Finance*, Vol. 47 No. 2, pp. 427-465, doi: 10.2307/2329112.

Farm Credit Administration (2014), "History of FCA and the FCS".

Federal Reserve Bank of St. Louis (2024), "FRED, economic data – Moody's seasoned Aaa corporate bond yield".

Francis, J.R., Khurana, I.K. and Pereira, R. (2005), "Disclosure incentives and effects on cost of capital around the world", *The Accounting Review*, Vol. 80 No. 4, pp. 1125-1162, doi: 10.2308/accr.2005.80.4.1125.

Gelman, A., Carlin, J.B., Stern, H.S., Dunson, D.B., Vehtari, A. and Rubin, D.B. (2013), *Bayesian Data Analysis*, 3rd ed., CRC Press, New York.

Grau, A. and Reig, A. (2020), "Operating leverage and the cost of debt in european agri-food firms", *Spanish Journal of Agricultural Research*, Vol. 18 No. 3, e0109, doi: 10.5424/sjar/2020183-16254.

Harl, N. (1990), *The Farm Debt Crisis of the 1980s*, Iowa State University Press, Iowa.

Hyytinen, A. and Pajarinen, M. (2007), "Is the cost of debt capital higher for younger firms?", *Scottish Journal of Political Economy*, Vol. 54 No. 1, pp. 55-71, doi: 10.1111/j.1467-9485.2007.00404.x.

Innes, R. (1990), "Imperfect information and the theory of government intervention in farm credit markets", *American Journal of Agricultural Economics*, Vol. 72 No. 3, pp. 761-768, doi: 10.2307/1243052.

Kuttner, K.N. (2018), "Outside the box: unconventional monetary policy in the great recession and beyond", *The Journal of Economic Perspectives*, Vol. 32 No. 4, pp. 121-146, doi: 10.1257/jep.32.4.121.

Lambert, R., Leuz, C. and Verrecchia, R.E. (2007), "Accounting information, disclosure, and the cost of capital", *Journal of Accounting Research*, Vol. 45 No. 2, pp. 385-420, doi: 10.1111/j.1475-679x.2007.00238.x.

Lintner, J. (1963), "The cost of capital and optimal financing of corporate growth", *The Journal of Finance*, Vol. 18 No. 2, pp. 292-310, doi: 10.2307/2977910.

Lintner, J. (1965), "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets", *The Review of Economics and Statistics*, Vol. 47 No. 1, pp. 13-37, doi: 10.2307/1924119.

Martin, S.J. and Clapp, J. (2015), "Finance for agriculture or agriculture for finance?", *Journal of Agarian Change*, Vol. 15 No. 4, pp. 549-559, doi: 10.1111/joac.12110.

Miles, J.A. and Ezzell, J.R. (1980), "The weighted average cost of capital, perfect capital markets, and project life: a clarification", *Journal of Financial and Quantitative Analysis*, Vol. 15 No. 3, p. 719, doi: 10.2307/2330405.

Miller, M.H. and Modigliani, F. (1961), "Dividend policy, growth and the valuation of shares", *Journal of Business*, Vol. 34 No. 4, pp. 411-433, doi: 10.1086/294442.

Miller, N., Ifft, J. and Mashange, G. (2024), "Farm loan interest rate trends by major lender groups", *Farmdoc Daily*, Vol. 14 No. 32, pp. 1-5.

Modigliani, F. and Miller, M.H. (1958), "The cost of capital, corporation finance and the theory of investment", *The American Economic Review*, Vol. 48 No. 3, pp. 261-297.

Myers, S.C. (1974), "Internations of corporate financing and investment decisions – implications for capital budgeting", *The Journal of Finance*, Vol. 29 No. 1, pp. 1-25, doi: 10.2307/2978211.

Schuh, G. (1974), "The exchange rate and U.S. agriculture", *American Journal of Agricultural Economics*, Vol. 56 No. 1, pp. 1-13, doi: 10.2307/1239342.

Schuh, G.E. (1976), "The new macroeconomics of agriculture", *American Journal of Agricultural Economics*, Vol. 58 No. 5, pp. 802-811, doi: 10.2307/1239976.

Sharpe, W. (1964), "Capital asset prices: a theory of market equilibrium under conditions of risk", *The Journal of Finance*, Vol. 19 No. 3, pp. 425-442, doi: 10.1111/j.1540-6261.1964.tb02865.x.

Sims, C. (1980), "Macroeconomics and reality", *Econometrica*, Vol. 48 No. 1, pp. 1-48, doi: 10.2307/1912017.

Solomon, E. (1963), *The Theory of Financial Management*, Columbia University Press, New York.

Stan Development Team (2022), "Stan".

Wharton School of Business, University of Pennsylvania (2024), "Wrds Wharton research data services".

Ylhäinen, I. (2017), "Life-cycle effects in small business finance", *Journal of Banking and Finance*, Vol. 77, pp. 176-196, doi: 10.1016/j.jbankfin.2017.01.008.

Zellner, A. (1985), "Bayesian econometrics", *Econometrica*, Vol. 53 No. 2, pp. 253-269, doi: 10.2307/1911235.

Corresponding author
Jaclyn D. Kropp can be contacted at: jkropp@ufl.edu

Appendices

A Lag structure

A.1 Imposing stationarity on the times series through it's roots
In this application, we impose stationarity on the autoregression coeffcients for the observed average interest rate. Specifically, we specify $\{\lambda_1, \lambda_2, \lambda_3\} \in [-0.95, 0.95]$. These coefficients can be used to specify the autoregressive parameters

$$(1+\lambda_1 L)(1+\lambda_2 L)(1+\lambda_3 L) = 1 + (\lambda_1+\lambda_2+\lambda_3)L + (\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)L^2 + \lambda_1\lambda_2\lambda_3 L^3 \tag{A.1}$$

where L denotes the lag function. From Equation (A.1), we define

$$\begin{aligned} \beta_1 &= \lambda_1+\lambda_2+\lambda_3 \\ \beta_2 &= \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3 \\ \beta_3 &= \lambda_1\lambda_2\lambda_3 \end{aligned} . \tag{A.2}$$

Hence, the constraints on λ_is guarantees that the time series specification is stationary.

A.2 Deriving the steady state solution
Building on Equation (A.2), the time-series representation in this study becomes

$$\bar{r}_t = \beta_0 + \beta_1\bar{r}_{t-1} + \beta_2\bar{r}_{t-2} + \beta_3\bar{r}_{t-3} + \epsilon_t. \tag{A.3}$$

Next, to develop the stationary solution of this representation, we assume that $\epsilon_t = 0$. In addition, we use the lag notation to modify Equation (A.3) (i.e. $L^i = \bar{r}_{t-i}$) Equation (A.3) can then be restated as

$$\left(1-\beta_1 L-\beta_2 L^2-\beta_3 L^3\right)\bar{r}_t = \beta_0 \tag{A.4}$$

Given the representation in Equation (A.4), we conjecture that a stead state solution exists (i.e. $\bar{r}_t = L\bar{r}_{t-1} = L^2\bar{r}_{t-2} = L^3\bar{r}_{t-3}$) so that the steady state solution becomes

$$(1-\beta_1-\beta_2-\beta_3)\bar{r}_t = \beta_0. \tag{A.5}$$

or the steady state solution exists

$$\bar{r}_t = \frac{\beta_0}{1-\beta_1-\beta_2-\beta_3}. \tag{A.6}$$

The steady state solution exists if $|1-\beta_1-\beta_2-\beta_3| \ll 1$. Mathematically, if this condition exists

$$\delta\bar{r}_t = \frac{\beta_0+\delta\beta_0}{1-\beta_1-\beta_2-\beta_3} \ll \infty \tag{A.7}$$

or any change in β_0 yields a bounded change in $\bar{r}_t$.

Reintroducing the error term (ϵ_t) and assuming that the error term is independently and identically distributed, Equation (A.7) becomes

$$\bar{r}_t = \frac{\beta_0+\epsilon_t}{1-\beta_1-\beta_2-\beta_3} \tag{A.8}$$

So if $\mathrm{E}(\epsilon_t) = 0$, the long-run steady state solution is identical to Equation (A.6). Expanding the original time-series function slightly as introduced in the text, we consider the time-series specification

$$\overline{r}_t = \beta_0 + \beta_1 \overline{r}_{t-1} + \beta_2 \overline{r}_{t-2} + \beta_3 \overline{r}_{t-3} + \alpha x_t + \epsilon_t. \tag{A.9}$$

where x_t is some exogenous variable. If x_t is a random series where its current value is not a function of lagged or future values, its steady state solution can be written following the general process used to construct Equation (A.8) as

$$\overline{r}_t = \frac{\beta_0 + \alpha x_t + \epsilon_t}{1 - \beta_1 - \beta_2 - \beta_3}. \tag{A.10}$$

Based on the formulation in Equation (A.10), we can derive the marginal effect of x_t on the steady state solution

$$\frac{\partial \overline{r}_t}{\partial x_t} = \frac{\alpha}{1 - \beta_1 - \beta_2 - \beta_3} \tag{A.11}$$

B Description of the Hamiltonian Markov Chain sampler

To develop the Hamiltonian Markov Chain sampler, we start with the augmented likelihood function (i.e. the likelihood function multiplied by a prior distribution) for the model implied by Equation (12)

$$\begin{aligned} g\left(\theta; \left\{ \overline{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stock} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) \ = \\ \left(\prod_{t=1}^{T} f_r\left(\overline{r}_t; \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks}, \theta\right) \right) \times f_N\left(\theta; \tilde{\theta}, \Sigma_\theta\right) \end{aligned} \tag{B.1}$$

where $\left\{ \overline{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stock} \right\}_{t=1}^{T}$ denotes the sample of dependant and independent variables, f_r () denotes the distribution of the observed imputed interest rates dependent on the observed data and a vector of parameters (θ), and $f_N\left(\theta, \tilde{\theta}, \Sigma_\theta\right)$ is the distribution of the parameters. We assume that f_r () follows the general model in Equation (12) such that the ϕ and α parameters are normally distribution about some mean and the residual is normally distributed with a mean of zero and an unknown variance. In a traditional (closed-form) Bayesian approach, we would define a proper distribution $(\tilde{g}())$ using the augmented distribution function presented in Equation (B.1).

Both the HMC and BUGS samplers start with an initial value and execute a number of draws. In our estimator we make 40,000 draws for each of six chains (replications) for each of the 15 ARMS states. Following conventions for both approaches the first half of each sample is discarded and the remaining sample is tested for information content and stationarity. In addition, we thin the sample, retaining only the fiftieth draw from each sample. Thinning helps reduce the potential for autocorrelation and non-stationarity within each sequence of draws. Hence, we generate 400 draws for each of six chains resulting in 2,400 draws in the posterior distribution for each state.

$$\begin{aligned} \tilde{g}\left(\theta; \left\{ \overline{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) = \\ \frac{g\left(\theta; \left\{ \overline{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right)}{\int_\theta g\left(\theta; \left\{ \overline{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) d\theta}. \end{aligned} \tag{B.2}$$

The Bayesian estimator would then be defined as the value of $\widehat{\theta}$ that minimizes the expected Bayesian loss function based on this proper posterior distribution. If we assume a simple quadratic loss function, the value that minimizes this loss function is simple the expected value based on the proper prior distribution

$$\widehat{\theta} = \int_{\theta} \theta \tilde{g}\left(\theta; \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_s^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) d\theta. \tag{B.3}$$

Conceptually, the first step in modern (numerical) approaches to Bayesian estimation involve approximating the expectation in Equation (B.3) using a simulation approach

$$\int_{\theta} \theta \tilde{g}\left(\theta; \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_s^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) d\theta \approx$$
$$\frac{1}{S}\sum_{s=1}^{S} \tilde{\theta}_s\left(\tilde{g}\left(\theta; \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, r_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right)\right) \tag{B.4}$$

where $\tilde{\theta}(\tilde{g}())$ denotes a random draw of the vector of parameters (θ) based on the distribution $\tilde{g}()$ (Gelman *et al*., 2013, p. 261-272).

A second modification in numerical Bayesian approaches involves the use of simpler forms of the distribution function that are easier to sample. Specifically, the original function $\tilde{g}()$ is approximated with a distribution function $q()$ which is proportion to the original distribution. In general, $q()$ is defined such that

$$\frac{q\left(\theta; \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right)}{\tilde{g}\left(\theta; \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_s^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right)} = M \tag{B.5}$$

where M is a constant.

Given this approximate, both HMC and BUGS use a Markov process to create a sample of parameter vectors that will be used as the posterior distribution. In general, the Markov process can be expressed as

$$q\left(\theta_k \middle| \theta_{k-1}, \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) =$$
$$\pi(\theta_k|\theta_{k=1}) q\left(\theta_{k-1} \middle| \theta_{k-2}, \left\{ \bar{r}_t, r_t^{Aaa}, \tilde{r}_t^{S\&P}, r_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) \tag{B.6}$$

where $\pi(\theta_k|\theta_{k-1})$ represents a transition probability distribution (or process). The BUGS sampler uses a combination of a Gibbs sampler and a Metropolis algorithm. The Gibbs sampler involves separating the parameters into groups and then drawing each group of parameters from $q(\theta_k|\theta_{k-1}, \cdots)$ using a combination of new and old parameters (i.e. a subset of θ_k and θ_{k-1}). The Metropolis algorithm modifies the probability step from θ_{k-1} to θ_k depending on the relative probability of the new point relative to the old point. If the probability of the new point is larger than the probability of the old point, the step is made. However, if the new point is less likely than the old point, the step is only made with a specified probability. In general, Markov processes are designed to increase the number of samples drawn from an area of parameter space that has non-zero likelihoods.

The Hamiltonian Markov Sampler implemented in MC-Stan uses a slightly different transition probability (Gelman *et al*., 2013, p. 300-305). As a starting point, the HMC sampler modifies Equation (B.6) slightly

$$q\left(\theta_k \middle| \theta_{k-1}, \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) =$$
$$\pi(\phi) q\left(\theta_{k-1} \middle| \theta_{k-2}, \left\{ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_t^{Ag\ Stocks} \right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right) \tag{B.7}$$

where $\pi(\phi)$ is an independent distribution of "ancillary" random variables which will be used to weight the step (i.e. the transition between θ_{k-1} and θ_k). As a simplification, $\phi \sim \mathrm{N}(, M_{jj})$ where M_{jj} is a "diagonal mass" matrix.

The HMC algorithm follows the general Metropolis formulation using two "half steps." The first step is to update the distribution function for ϕ^k. This update involves "leapfrog steps" based on a scaling factor ξ which is updated L times. This leapfrogging is based on the logarithmic gradient of the prior distribution

$$\phi_j^* = \phi_j + \frac{1}{2}\xi\left(\frac{d\ln\left(q\left(\theta\middle|\left\{\ \bar{r}_t, \tilde{r}_t^{Aaa}, \tilde{r}_t^{S\&P}, \tilde{r}_T^{Ag\ \ Stocks}\right\}_{t=1}^{T}, \tilde{\theta}, \Sigma_\theta\right)\right)}{\theta_j}\right) \tag{B.8}$$

where ϕ_j is a draw from the $\mathrm{N}(0, M_{jj})$ distribution based on the last iteration. The basic concept is to "speed up" or make larger steps when the draws is from a less informative area of the distribution. In addition, the derivative of the logarithm of the probability density function tends to push the values back toward the center. If the derivative is positive, the starting point lies to the left of the distribution and the leapfrog effect is positive. If the derivative is negative, the starting point lies to the right of the distribution and the leapfrog effect is negative. The new value of the parameter is then computed in the second "half-step"

$$\theta_k = \theta_{k-1} + \xi M^{-1}\phi^* \tag{B.9}$$

and the values of ϕ_j^k are updated.

Table A1. Standard industrial classification codes for agricultural stocks

Number	Code	Number	Code	Number	Code	Number	Code
1	111	31	259	61	2026	91	2084
2	112	32	271	62	2032	92	2085
3	115	33	272	63	2033	93	2087
4	116	34	273	64	2034	94	2091
5	119	35	279	65	2035	95	2092
6	131	36	291	66	2037	96	2095
7	132	37	711	67	2038	97	2096
8	133	38	721	68	2041	98	2098
9	134	39	722	69	2043	99	2099
10	139	40	723	70	2044	100	2111
11	161	41	724	71	2045	101	2121
12	171	42	741	72	2046	102	2131
13	172	43	742	73	2047	103	2141
14	173	44	751	74	2048	104	2211
15	174	45	752	75	2051	105	2221
16	175	46	761	76	2052	106	2231
17	179	47	762	77	2053	107	2241
18	181	48	781	78	2061	108	2261
19	182	49	782	79	2062	109	2299
20	191	50	783	80	2063	110	2873
21	211	51	811	81	2064	111	2874
22	212	52	831	82	2066	112	2875
23	213	53	851	83	2067	113	2879
24	214	54	2011	84	2068	114	3199
25	219	55	2013	85	2074	115	3523
26	241	56	2015	86	2075	116	5083
27	251	57	2021	87	2076		
28	252	58	2022	88	2077		
29	253	59	2023	89	2079		
30	254	60	2024	90	2083		

Source(s): Table created by authors

Table A2. RStan output

Variable	Mean	SE mean	Std dev	Quantiles 0.025	0.250	0.500	0.975	0.975	N eff	R hat
Arkansas										
a0	−0.01902	0.00037	0.01707	−0.05286	−0.03057	−0.01912	−0.00787	0.01476	2176.6	1.00203
a1	0.35966	0.00406	0.19279	−0.03264	0.23297	0.36408	0.48710	0.73619	2253.3	1.00203
a2	0.05968	0.00104	0.05154	−0.04406	0.02669	0.05956	0.09369	0.15897	2464.5	1.00131
a3	−0.07091	0.00111	0.05647	−0.18346	−0.10864	−0.07061	−0.03432	0.04618	2578.1	0.99906
b0	0.01222	0.00019	0.00928	−0.00603	0.00581	0.01209	0.01844	0.03071	2285.3	1.00046
b1	0.95365	0.00239	0.11183	0.72770	0.87826	0.95375	1.03116	1.16468	2197.2	1.00034
b2	−0.00768	0.00143	0.07110	−0.13518	−0.05693	−0.01217	0.03867	0.13704	2487.5	0.99874
b3	−0.11162	0.00177	0.08408	−0.29117	−0.16816	−0.10101	−0.04217	0.00216	2254.2	0.99986
σ^2	0.00627	0.00001	0.00070	0.00511	0.00576	0.00621	0.00672	0.00781	2272.0	1.00235
1 – b1 – b2 – b3	0.16564	0.00212	0.10187	−0.03266	0.09755	0.16481	0.23582	0.36278	2317.4	1.00000
California										
a0	−0.02260	0.00025	0.01262	−0.04713	−0.03120	−0.02252	−0.01453	0.00233	2506.9	1.00034
a1	0.41923	0.00309	0.15438	0.12101	0.31826	0.42106	0.52323	0.72494	2490.1	1.00054
a2	−0.07031	0.00070	0.03495	−0.14005	−0.09358	−0.07123	−0.04705	−0.00064	2469.8	1.00040
a3	0.00024	0.00091	0.04554	−0.09077	−0.02994	0.00058	0.02984	0.09002	2499.3	0.99943
b0	0.00939	0.00012	0.00598	−0.00221	0.00549	0.00942	0.01336	0.02115	2610.7	0.99972
b1	0.85110	0.00194	0.09721	0.67905	0.78192	0.84693	0.91541	1.04747	2509.0	1.00040
b2	0.04883	0.00122	0.06126	−0.07120	0.00565	0.04934	0.09415	0.16296	2526.6	0.99891
b3	−0.04123	0.00100	0.04978	−0.17520	−0.06498	−0.02541	−0.00306	0.00951	2469.2	0.99962
σ^2	0.14130	0.00137	0.07021	0.00382	0.09658	0.13989	0.18884	0.27819	2627.4	0.99961
1 – b1 – b2 – b3	0.00510	0.00001	0.00057	0.00411	0.00469	0.00505	0.00544	0.00639	2298.2	1.00023
Florida										
a0	−0.00787	0.00019	0.00936	−0.02637	−0.01388	−0.00769	−0.00166	0.01105	2396.1	1.00093
a1	0.23870	0.00283	0.13832	−0.02961	0.14821	0.23528	0.32659	0.52164	2392.3	1.00084
a2	0.04283	0.00095	0.04330	−0.04542	0.01424	0.04304	0.07111	0.12568	2056.0	1.00069
a3	−0.04744	0.00087	0.03917	−0.12377	−0.07335	−0.04812	−0.02129	0.02979	2049.1	1.00034
b0	0.00091	0.00009	0.00433	−0.00714	−0.00197	0.00081	0.00375	0.00940	2351.8	1.00073

(*continued*)

Table A2. Continued

Variable	Mean	SE mean	Std dev	Quantiles 0.025	0.250	0.500	0.975	0.975	N eff	R hat
b1	0.85972	0.00172	0.08077	0.73045	0.79936	0.85098	0.91177	1.03939	2208.7	0.99849
b2	0.11732	0.00114	0.05469	0.00151	0.07974	0.12470	0.15995	0.20303	2296.3	0.99985
b3	−0.01580	0.00074	0.03511	−0.11259	−0.02490	−0.00341	0.00616	0.01559	2261.1	0.99959
σ^2	0.03876	0.00118	0.05727	−0.07171	−0.00007	0.03842	0.07709	0.15377	2370.4	1.00035
1 – b1 – b2 – b3	0.00652	0.00001	0.00072	0.00532	0.00599	0.00645	0.00696	0.00810	2328.3	1.00010
Georgia										
a0	−0.05797	0.00033	0.01645	−0.08961	−0.06934	−0.05793	−0.04710	−0.02537	2462.5	1.00057
a1	0.88726	0.00425	0.21540	0.47213	0.74271	0.88149	1.03370	1.30655	2568.1	0.99992
a2	0.06956	0.00108	0.05312	−0.03247	0.03327	0.06848	0.10553	0.17610	2422.9	1.00022
a3	−0.14768	0.00089	0.04442	−0.23517	−0.17601	−0.14784	−0.11934	−0.06480	2480.2	1.00008
b0	0.03850	0.00020	0.00947	0.01953	0.03227	0.03868	0.04486	0.05649	2307.8	1.00015
b1	0.93738	0.00205	0.10048	0.72487	0.87575	0.93988	1.00644	1.12266	2402.1	0.99827
b2	−0.17153	0.00125	0.06170	−0.28571	−0.21389	−0.17388	−0.13392	−0.04127	2422.8	0.99979
b3	−0.21693	0.00164	0.08043	−0.37041	−0.27174	−0.21872	−0.16419	−0.05782	2393.4	0.99874
σ^2	0.00683	0.00002	0.00077	0.00552	0.00630	0.00675	0.00731	0.00855	2326.3	0.99932
1 – b1 – b2 – b3	0.45107	0.00220	0.10549	0.23920	0.38188	0.45259	0.52290	0.65468	2303.5	1.00015
Illinois										
a0	−0.05655	0.00039	0.01868	−0.09323	−0.06890	−0.05667	−0.04403	−0.01994	2294.4	0.99902
a1	0.68981	0.00451	0.21409	0.28545	0.55125	0.68804	0.83009	1.10940	2251.9	0.99895
a2	0.09023	0.00091	0.04168	0.00976	0.06152	0.09043	0.11802	0.16797	2114.5	0.99886
a3	−0.05242	0.00103	0.04833	−0.14571	−0.08403	−0.05346	−0.01997	0.04201	2203.8	0.99939
b0	0.02467	0.00018	0.00864	0.00771	0.01894	0.02458	0.03049	0.04175	2261.1	0.99944
b1	0.95252	0.00229	0.11176	0.72955	0.87667	0.95774	1.02991	1.16076	2377.0	1.00062
b2	−0.07729	0.00128	0.06131	−0.18614	−0.11909	−0.08166	−0.03972	0.05635	2300.2	1.00057
b3	−0.16055	0.00164	0.08279	−0.32553	−0.21812	−0.16020	−0.10039	−0.00558	2544.5	1.00090
σ^2	0.00527	0.00001	0.00060	0.00428	0.00483	0.00521	0.00563	0.00663	2505.2	1.00080
1 – b1 – b2 – b3	0.28533	0.00197	0.09328	0.10622	0.22443	0.28268	0.34830	0.46997	2236.8	0.99931

(*continued*)

Table A2. Continued

Variable	Mean	SE mean	Std dev	Quantiles 0.025	0.250	0.500	0.975	0.975	N eff	R hat
Indiana										
a0	−0.03885	0.00046	0.02212	−0.08208	−0.05373	−0.03908	−0.02428	0.00452	2319.1	0.99885
a1	0.53577	0.00521	0.25034	0.05410	0.36312	0.53766	0.70381	1.02991	2312.1	0.99914
a2	0.05803	0.00089	0.04271	−0.02210	0.02975	0.05726	0.08569	0.14416	2280.3	0.99895
a3	−0.06365	0.00096	0.04439	−0.15064	−0.09396	−0.06416	−0.03286	0.02418	2141.3	0.99890
b0	0.01687	0.00023	0.01077	−0.00530	0.00978	0.01711	0.02377	0.03789	2163.7	0.99976
b1	0.98013	0.00232	0.11115	0.75798	0.90355	0.98260	1.05847	1.18964	2296.9	0.99938
b2	−0.04214	0.00142	0.06878	−0.17265	−0.08717	−0.04473	−0.00021	0.10154	2347.1	1.00031
b3	−0.14067	0.00173	0.08481	−0.31027	−0.19965	−0.13858	−0.07657	−0.00085	2394.3	1.00012
σ^2	0.00528	0.00001	0.00058	0.00426	0.00487	0.00524	0.00564	0.00655	2240.0	1.00006
1 – b1 – b2 – b3	0.20269	0.00255	0.11521	−0.03673	0.12820	0.20159	0.28042	0.43409	2045.2	0.99970
Iowa										
a0	−0.09145	0.00041	0.01940	−0.12941	−0.10441	−0.09141	−0.07885	−0.05283	2235.1	1.00595
a1	1.17258	0.00499	0.22776	0.72003	1.02192	1.17577	1.32348	1.60242	2085.1	1.00671
a2	0.14957	0.00070	0.03228	0.08844	0.12821	0.14915	0.17141	0.21370	2097.2	1.00421
a3	−0.16713	0.00094	0.04418	−0.25417	−0.19735	−0.16678	−0.13834	−0.07883	2200.9	1.00162
b0	0.04353	0.00018	0.00891	0.02634	0.03757	0.04358	0.04944	0.06004	2351.8	1.00372
b1	0.90130	0.00225	0.09150	0.71440	0.84166	0.90323	0.96123	1.07523	1655.9	1.01092
b2	−0.19153	0.00121	0.05759	−0.30047	−0.23183	−0.19357	−0.15347	−0.06894	2266.7	0.99933
b3	−0.21069	0.00158	0.07363	−0.35630	−0.25908	−0.21190	−0.16178	−0.06773	2183.0	1.00358
σ^2	0.00454	0.00001	0.00052	0.00370	0.00417	0.00449	0.00485	0.00570	1467.3	1.01320
1 – b1 – b2 – b3	0.50092	0.00200	0.09689	0.31024	0.43564	0.50214	0.56493	0.68624	2339.6	1.00392
Kansas										
a0	−0.02798	0.00041	0.02002	−0.06843	−0.04150	−0.02755	−0.01456	0.01138	2384.1	0.99957
a1	0.50305	0.00506	0.25574	0.00782	0.32817	0.50163	0.67365	1.00228	2549.6	0.99909
a2	−0.00745	0.00090	0.04343	−0.09582	−0.03541	−0.00661	0.02031	0.07844	2338.6	0.99994
a3	−0.05450	0.00105	0.05248	−0.15798	−0.08841	−0.05505	−0.02149	0.04951	2515.1	0.99922
b0	0.01435	0.00020	0.00966	−0.00471	0.00780	0.01438	0.02064	0.03350	2276.7	0.99999

(*continued*)

Table A2. Continued

Variable	Mean	SE mean	Std dev	Quantiles 0.025	0.250	0.500	0.975	0.975	N eff	R hat
b1	0.92490	0.00205	0.10387	0.72564	0.85468	0.92666	0.99362	1.12988	2575.3	0.99901
b2	−0.01360	0.00133	0.06602	−0.13436	−0.05904	−0.01618	0.02960	0.12123	2451.2	1.00050
b3	−0.09300	0.00143	0.06860	−0.24156	−0.14041	−0.08587	−0.03572	0.00138	2315.0	0.99996
σ^2	0.00547	0.00001	0.00060	0.00444	0.00504	0.00541	0.00583	0.00685	2324.0	0.99935
1 – b1 – b2 – b3	0.18170	0.00213	0.10509	−0.02829	0.11064	0.18155	0.25112	0.38495	2429.9	0.99969
Minnesota										
a0	−0.03848	0.00049	0.02331	−0.08183	−0.05455	−0.03848	−0.02259	0.00747	2258.3	0.99844
a1	0.59547	0.00544	0.25600	0.09367	0.42541	0.59751	0.76973	1.10294	2213.3	0.99893
a2	0.06057	0.00121	0.05652	−0.05264	0.02266	0.06052	0.09758	0.16918	2181.4	0.99885
a3	−0.07767	0.00113	0.05552	−0.18659	−0.11513	−0.07846	−0.04011	0.02825	2434.8	0.99893
b0	0.01658	0.00022	0.01071	−0.00471	0.00943	0.01653	0.02373	0.03743	2300.1	0.99877
b1	0.88917	0.00233	0.11076	0.67042	0.81295	0.88848	0.96117	1.11525	2264.0	1.00069
b2	−0.02081	0.00167	0.07924	−0.16603	−0.07410	−0.02572	0.03167	0.14476	2238.5	0.99802
b3	−0.08686	0.00155	0.07665	−0.26337	−0.13290	−0.07234	−0.02167	0.00509	2431.5	0.99963
σ^2	0.00642	0.00001	0.00070	0.00521	0.00593	0.00637	0.00685	0.00794	2423.4	1.00111
1 – b1 – b2 – b3	0.21850	0.00244	0.11651	−0.00758	0.14236	0.22163	0.29691	0.44337	2276.9	0.99895
Missouri										
a0	−0.01946	0.00040	0.01933	−0.05790	−0.03245	−0.01939	−0.00672	0.01734	2373.0	1.00072
a1	0.35805	0.00442	0.21801	−0.06629	0.21066	0.35622	0.49785	0.79799	2429.0	1.00001
a2	0.07122	0.00094	0.04472	−0.01852	0.04185	0.07236	0.10034	0.15907	2254.2	1.00114
a3	−0.06738	0.00108	0.05146	−0.16773	−0.10069	−0.06656	−0.03333	0.03555	2252.6	0.99968
b0	0.01040	0.00020	0.00977	−0.00891	0.00396	0.01039	0.01688	0.02953	2366.4	1.00128
b1	0.94849	0.00226	0.11287	0.72882	0.87065	0.94783	1.02484	1.18051	2496.7	1.00169
b2	0.00774	0.00151	0.07254	−0.12443	−0.04114	0.00344	0.05515	0.15854	2320.7	1.00039
b3	−0.09957	0.00165	0.08032	−0.27915	−0.15307	−0.08881	−0.03178	0.00453	2378.4	1.00144
σ^2	0.00537	0.00001	0.00059	0.00437	0.00496	0.00532	0.00573	0.00667	2374.8	0.99899
1 – b1 – b2 – b3	0.14334	0.00216	0.10588	−0.06193	0.07294	0.14273	0.21515	0.35025	2393.5	1.00116

(*continued*)

Table A2. Continued

Variable	Mean	SE mean	Std dev	Quantiles 0.025	0.250	0.500	0.975	0.975	N eff	R hat
Nebraska										
a0	−0.02173	0.00049	0.02185	−0.06476	−0.03603	−0.02141	−0.00719	0.01931	1985.6	1.00064
a1	0.60860	0.00551	0.24750	0.13135	0.44688	0.59997	0.76975	1.10604	2015.8	0.99872
a2	0.08605	0.00125	0.06330	−0.04137	0.04450	0.08640	0.12749	0.21021	2557.9	0.99886
a3	−0.18175	0.00130	0.06491	−0.30691	−0.22660	−0.18202	−0.13583	−0.05451	2479.2	0.99918
b0	0.01189	0.00021	0.00948	−0.00622	0.00558	0.01178	0.01813	0.03092	2010.6	0.99977
b1	0.90541	0.00211	0.10130	0.70109	0.83831	0.90635	0.97145	1.10390	2300.8	0.99837
b2	−0.01708	0.00149	0.07007	−0.14514	−0.06553	−0.02013	0.02683	0.12926	2216.6	0.99953
b3	−0.08500	0.00151	0.07294	−0.25810	−0.13156	−0.07042	−0.02244	0.00324	2334.1	0.99949
$sigma^2$	0.00578	0.00001	0.00064	0.00469	0.00533	0.00573	0.00615	0.00725	2285.5	1.00062
1 – b1 – b2 – b3	0.19666	0.00230	0.10235	0.00100	0.12953	0.19362	0.26665	0.39839	1977.7	0.99906
North Carolina										
a0	−0.02995	0.00034	0.01672	−0.06490	−0.04058	−0.02952	−0.01909	0.00280	2419.0	0.99913
a1	0.62132	0.00365	0.18257	0.27885	0.50005	0.62049	0.73863	1.00093	2504.6	0.99903
a2	−0.01483	0.00086	0.04083	−0.09521	−0.04203	−0.01462	0.01192	0.06312	2277.5	1.00040
a3	−0.10637	0.00098	0.04765	−0.19919	−0.13806	−0.10780	−0.07458	−0.00981	2378.5	1.00083
b0	0.01549	0.00016	0.00789	0.00065	0.01019	0.01553	0.02067	0.03144	2409.5	1.00055
b1	0.77267	0.00195	0.09321	0.60669	0.70574	0.77051	0.83474	0.96016	2285.3	0.99950
b2	0.02580	0.00144	0.06842	−0.10833	−0.02216	0.03005	0.07662	0.14281	2241.8	1.00003
b3	−0.03215	0.00089	0.04090	−0.13995	−0.05043	−0.01784	−0.00184	0.00802	2133.3	0.99848
σ^2	0.00613	0.00001	0.00068	0.00495	0.00563	0.00605	0.00657	0.00756	2413.2	0.99919
1 – b1 – b2 – b3	0.23367	0.00180	0.08856	0.06422	0.17254	0.23343	0.29203	0.40633	2414.1	1.00055
Texas										
a0	−0.01614	0.00040	0.01992	−0.05437	−0.02933	−0.01593	−0.00252	0.02292	2438.1	1.00167
a1	0.33697	0.00458	0.22556	−0.11514	0.18106	0.34145	0.48660	0.78391	2426.3	1.00143
a2	0.04481	0.00161	0.07861	−0.10528	−0.00710	0.04334	0.09712	0.20098	2387.9	0.99942
a3	−0.03727	0.00146	0.07153	−0.17424	−0.08596	−0.03714	0.01179	0.10291	2393.1	0.99972
b0	0.00565	0.00016	0.00797	−0.00996	0.00037	0.00575	0.01081	0.02157	2340.8	1.00067

(*continued*)

Table A2. Continued

Variable	Mean	SE mean	Std dev	Quantiles 0.025	0.250	0.500	0.975	0.975	N eff	R hat
b1	0.87516	0.00191	0.09268	0.70010	0.81135	0.87503	0.93635	1.06657	2355.3	0.99909
b2	0.06608	0.00139	0.06623	−0.06384	0.01815	0.06569	0.11418	0.18941	2278.3	1.00006
b3	−0.03146	0.00088	0.04259	−0.14355	−0.05011	−0.01639	−0.00054	0.01279	2361.8	1.00247
σ^2	0.00705	0.00002	0.00080	0.00573	0.00650	0.00698	0.00751	0.00883	2294.5	1.00163
1 – b1 – b2 – b3	0.09022	0.00188	0.09111	−0.08724	0.02939	0.09035	0.15115	0.27206	2356.3	1.00085
Washington										
a0	−0.03871	0.00025	0.01183	−0.06213	−0.04654	−0.03877	−0.03082	−0.01604	2271.2	0.99989
a1	0.56702	0.00318	0.15131	0.27602	0.47088	0.56479	0.67004	0.86052	2258.5	1.00011
a2	0.11844	0.00088	0.04031	0.03829	0.09235	0.11831	0.14481	0.19792	2120.2	1.00398
a3	−0.06018	0.00065	0.02916	−0.11699	−0.07968	−0.06063	−0.04029	−0.00301	1999.1	1.00383
b0	0.01691	0.00013	0.00617	0.00489	0.01279	0.01674	0.02114	0.02901	2188.4	1.00099
b1	0.82176	0.00191	0.09249	0.64254	0.75909	0.82123	0.87940	1.01912	2334.2	0.99894
b2	0.00230	0.00136	0.06530	−0.11154	−0.04628	−0.00154	0.04906	0.13349	2311.0	1.00109
b3	−0.04544	0.00110	0.04955	−0.17893	−0.07088	−0.03026	−0.00569	0.00573	2030.8	0.99925
σ^2	0.00515	0.00001	0.00057	0.00423	0.00474	0.00510	0.00549	0.00644	2268.7	1.00073
1 – b1 – b2 – b3	0.22138	0.00153	0.07128	0.08290	0.17527	0.21927	0.27035	0.35922	2169.7	1.00094
Wisconsin										
a0	−0.10992	0.00065	0.02970	−0.16720	−0.12908	−0.11056	−0.09078	−0.05278	2086.3	0.99907
a1	1.24037	0.00670	0.29677	0.65071	1.04593	1.24353	1.43178	1.82635	1960.8	0.99920
a2	−0.00964	0.00085	0.03989	−0.09229	−0.03438	−0.00907	0.01691	0.06549	2212.4	0.99901
a3	−0.05650	0.00087	0.03970	−0.13525	−0.08223	−0.05705	−0.03162	0.02244	2072.2	0.99834
b0	0.05982	0.00033	0.01479	0.03065	0.05056	0.05995	0.06980	0.08737	2068.0	0.99907
b1	0.88867	0.00199	0.09255	0.70089	0.82650	0.89198	0.95283	1.06221	2163.7	1.00071
b2	−0.28780	0.00194	0.08825	−0.44651	−0.34958	−0.28878	−0.23357	−0.10878	2060.7	0.99887
b3	−0.27509	0.00206	0.09079	−0.44512	−0.33835	−0.27666	−0.21632	−0.08967	1949.9	0.99937
σ^2	0.00495	0.00001	0.00058	0.00396	0.00455	0.00489	0.00528	0.00627	2229.0	1.00045
1 – b1 – b2 – b3	0.67422	0.00355	0.15948	0.35371	0.57623	0.67540	0.78432	0.97311	2020.6	0.99913

Source(s): Table created by authors